Testing the Evolution of Prices in Capital-Resource Economies under Recursive Preferences [*]

Johnson Kakeu[†] Pierre Nguimkeu[‡]

May 2022

Abstract

While understanding investors' behavior is critical for managing and predicting natural resource pricing dynamics, empirical studies have rejected intertemporal separability of investors' preferences in capital-resources economies. We present a dynamic model of exhaustible resource extraction to investigate the role of recursive investors' preferences in driving the evolution of natural resource pricing. An equilibrium condition emerges showing that under such preferences the growth rate of exhaustible resource prices must depend on both the available stock of resource and the stock of capital used to produce goods. This extends the basic Hotelling's pricing rule and provides a ground for a simple empirical test of the underlying investors' behavior. Using aggregate time series data for crude oil, our empirical results confirm nonseparability by showing that oil and capital stocks are significant determinants of oil prices. Policy implications for energy price forecasting are discussed.

Keywords: Recursive utility, Oil prices, Hotelling's rule, Regression analysis.

JEL Classifications: Q31, D60, C51, C52.

---

[*] Prior versions of this paper have circulated under the title "Capital-Resource Economies under Recursive Preferences". We thank Spencer Banzhaf, Robert Cairns, James Hamilton, Garth Heutel, Stephen Holland, Anastasios Karantounias, Martin Smith, Henry Thompson, participants of the Southern Economic Association meeting, the Canadian Economic Association meeting, the Western Economic Association International, the European Association of Environmental and Resource Economists meeting, and seminar participants at the Federal Reserve Bank of Atlanta, Morehouse College and Georgia State University.

[†] Department of Economics, University of Prince Edward Island, Prince Edward Island, Canada, C1A 4P; Email: jkakeu@upei.ca

[‡] Georgia State University, Andrew Young School of Policy Studies, 14 Marietta Street, NW, Atlanta, GA 30303, USA. Email: nnguimkeu@gsu.edu

1 Introduction

How does resource pricing in capital-resources economies adjust to the structure of intertemporal preferences ? When the structure of intertemporal preferences is assumed to be time-additive separable, the evolution of the shadow price of resource in the equilibrium is governed by the basic *Hotelling's rule*, which states that prices must rise at the rate of interest in nonrenewable resource markets. However, the empirical validity of the Hotelling's rule has not been strongly supported by the data (Halvorsen and Smith, 1991; Krautkraemer, 1998; Hamilton, 2009; Gaudet, 2007), while the intertemporal separability has increasingly been recognized as a strong assumption in many theoretical papers (Samuelson, 1937; Epstein, 1987; Skiadas, 2007). Yet, the traditional time-additive separability assumption has remained popular, mainly because of its great simplification of the analysis of intertemporal choice (Boyd III, 1990). However, as pointed out by many authors, the structure of social preferences needs not be intertemporally separable (Browning, 1991; Frederick et al., 2002; Epstein and Allan, 1983; Hertzendorf, 1995; Peltola and Knapp, 2001). Understanding investors' behavior is critical for the management of natural resources and the prediction of their pricing dynamics. For a detailed discussion regarding separability and functional structure, see the substantial body of work done by Gorman (1968); Mak (1986); Streufert (1995); Koopmans (1986).

This paper explores the implications of time nonseparability for natural resource management by developing a dynamic model of exhaustible resource extraction when the social planner is endowed with recursive preferences. The latter is a more flexible class of utility functionals which allows for a wide range of attitudes towards the entire future. We show that using recursive preferences in a capital-resource economy one can generate predictions that can be reconciled with the patterns observed in the data. Specifically, we derive a pricing rule for nonrenewable resources which predicts that the future resource price is the current spot price adjusted by a factor that depends on both the remaining stock of resource and the stock of man-made capital in the economy, and we confront this prediction with the data. The recursive preferences assumption underlying our equilibrium conditions accommodates the time-additive separable preferences underlying the classical Hotelling's principle as a special case, and highlights both the remaining stock of resources and the stock of capital as structural factors driving resource prices. Using annual data of the world's oil proven reserves and extractions, real oil prices and real stock of capital per capita, a proxy for the remaining oil stock series is constructed and an econometric analysis is conducted to examine the empirical content of the model's equilibrium condition. A comparison with the basic Hotelling rule of resource extraction is also provided.

While the lack of empirical support for Hotelling's model and some of its variants remains an empirical puzzle, several papers have attempted to resolve it in various ways (see, e.g. Livernois (2009) for a detailed review). One approach that has been investigated early on in the literature is to modify the Hotelling's rule to incorporate the degradation effect of extraction costs and technological change. The former forces scarcity rents to rise at a rate lower than the interest rate while the latter produces a countervailing effect (Slade, 1982; Schmidt, 1988; Livernois and Martin, 2001). However, subsequent research has cast doubt on the statistical robustness of these modifications. Another approach adjusts the Hotelling's rule to incorporate the role of risk in nonrenewable resource assets (Young, 1992; Slade and Thille, 1997; Gaudet, 2007). The intuition is that if the risk premium is sufficiently negative, the risk-adjusted Hotelling's rule could imply flat or even decreasing scarcity rent. However, the estimation often gives implausible results and statistical tests are found to have little power. Other approaches are based on a direct modification of the empirical Hotelling equation (Geoffrey and Barrow, 1981; Heal and Barrow, 1980; Agbeyegbe, 1989) or a modified Hotelling rule for stumpage prices that accounts for the opportunity cost of the land occupied by standing timber (Livernois et al., 2006).

Our paper therefore fits in this extensive literature that endeavors to understand the evolution of resource prices from a theoretical standpoint that has empirical content. However, unlike existing approaches, ours uses a recursive utility framework which accounts for attitude towards long-run future prospects in making decisions (Howitt et al., 2005; Knapp and Olson, 1996) and for which some of the approaches discussed above are special cases. In particular, with this form of time nonseparability, the current utility depends on both the current consumption and the future utility index which captures future consumption prospects. This therefore has important implications for optimal allocations if consumption depends on a commodity that depletes irreversibly over time. Empirical evidence with crude oil supports intertemporal nonseparability of preferences by showing that oil and capital stocks are relevant and statistically significant determinants of the relative changes in crude oil prices, with substantial explanatory power. These findings have interesting policy implications for food security and long-term forecasting of energy prices.

The rest of the paper is organized as follows. In Section 2, we present the theoretical framework and derive the dynamic behaviour of resource prices under recursive preferences. Section 3 provides an econometric analysis where our theory is tested with the data. Implications for resource policy are also discussed. Section 4 draws concluding remarks, and some technical material are given in the appendix.

2 Theoretical Framework

In this section, we derive our theoretical pricing rule under recursive preferences and show how the Hotelling's rule arises as a special case. Following Dasgupta and Heal (1974), Solow (1974) and Stiglitz (1974), we assume a simple economy in which there are two stock variables consisting of an exhaustible resource X (which we will often refer to as oil to fix ideas) and a produced or man-made capital K. The initial stock of resource $X_0 > 0$ is exhausted over the infinite time horizon and the initial endowment of man-made capital is $K_0 > 0$. Denote by $X(t)$ the stock of the exhaustible resource available at time t, then the dynamics of the exhaustible resource depletion is given by:

$$\dot{X}(t) = -x(t), \tag{1}$$

where $x(t)$ is the extraction flow rate at time t.[1] The technology is represented by a production function, $F(K(t), x(t))$, satisfying standard Inada conditions, that is, $F_K > 0$, $F_x > 0$, $F_{KK} < 0$, $F_{xx} < 0$ and $F(K, 0) = F(0, x) = 0$. Furthermore, for $K > 0$, $\lim_{x \to 0} F_x(K, x) = \infty$; and for $x > 0$, $\lim_{K \to 0} F_K(K, x) = \infty$. The dynamics of capital accumulation is given by:

$$\dot{K}(t) = F(K(t), x(t)) - c(t) - \delta K(t), \tag{2}$$

where $c(t)$ is the consumption at time t, and δ is the exogenous rate of physical depreciation of man-made capital. A more plausible interpretation of equation (2) is that the man-made capital is made from natural resources and not the converse.

2.1 Equilibrium Conditions under Recursive Preferences

In this framework the social planner has a recursive utility *à la* Epstein (1987). Recursive preferences focus on the trade-off between current-period utility and the utility to be derived

---

[1] In this notation, a single dot over a variable signifies its first derivative with respect to time.

from all future periods. For a given consumption process, denote by $J(t)$ the continuation utility attributable to a future consumption stream $\{c(s) : s > t\}$.[2] Recursive utility is represented by a function called an aggregator, $f(c(t); J(t))$, expressing current utility of a consumption path as a function of current consumption and the future utility derived from the remaining periods' consumption.[3]

The social planner chooses all quantities directly, taking all relevant information into account. At each time t the social planner decides how much to consume $c(t)$. Since the produced capital $K(t)$ depreciates at a constant rate δ, the social planner solves the following infinite horizon problem:

$$J(K_0, X_0) = \max_{c(t), x(t), t \in [0,\infty]} \int_0^\infty f(c(t), J(t))dt, \tag{3}$$

subject to the dynamics of the resource and the man-made capital:

$$\begin{aligned} \dot{X}(t) &= -x(t), & (4)\\ \dot{K}(t) &= F(K(t), x(t)) - c(t) - \delta K(t), & (5)\\ X(0) &= X_0 > 0, \quad K(0) = K_0 > 0. & (6) \end{aligned}$$

Given that $J(t) = J(K(t), X(t))$ is the value function, maximized utility, of the representative consumer in state $(K(t), X(t))$ at time t, then the shadow price $J_X(t)$ of the resource is the increase in social well-being that would be enjoyed if an additional unit of the resource was made available without a cost (i.e. the social worth of a marginal unit of the resource).

As shown in the Appendix, the dynamics of the shadow price of the resource is given by:

$$\frac{\dot{J}_X(t)}{J_X(t)} = -f_J(c(t); J(K(t), X(t))). \tag{7}$$

Equation (7) is our pricing rule which states that the growth rate of the shadow price of the resource accounts for the depletion of the exhaustible resource and the accumulation of the produced capital. It shows that both the consumption level $c(t)$, the current size of the natural resource stock $X(t)$ and the current size of the man-made capital $K(t)$ provide a sufficient statistic for the optimal rate of change in the shadow price of the marginal unit of exhaustible natural resource stock. Hence, one cannot in general formulate an hypothesis

---

[2] $J(\cdot)$ must satisfy a transversality condition of the form $\lim_{t\to\infty} e^{-\nu t} (|J(t)|) = 0$, for a suitable constant $\nu > 0$ (Duffie and Epstein, 1992, p.388).

[3] In other words, the aggregator function $f(c(t); J(t))$ combines present consumption and future utility to yield current utility of a consumption path. The aggregator function completely characterizes preferences. With this aggregator approach, the model is exempt of time inconsistencies.

about the direction of the growth rate of the shadow price of exhaustible natural resource stock without further assumptions on the aggregator of the underlying recursive utility.

The basic Hotelling rule can be obtained as a special case of the pricing rule (7). This corresponds to the case where the aggregator takes the form $f(c, J) = U(c) - rJ$ so that the discount rate $-f_J(c(t), J(K(t), X(t))) = r$ does not explicitly depend on the future utility index $J(K(t), X(t))$. Hence, the pricing rule reduces to the basic Hotelling rule given by:

$$\frac{\dot{J}_X(t)}{J_X(t)} \;=\; r. \tag{8}$$

Thus, the general formulation of the exhaustible natural resource economy under recursive utility considered here cannot rationalize the data less accurately than the time-additive utility, since the latter is a special case of the former. This also suggests that empirical studies investigating the structure of intertemporal preferences for informing exhaustible natural resource policy is a worthwhile axis of research.

2.2 Practical Considerations

For a practical assessment of dynamic issues implied by the use of continuous time recursive preferences, we make use of a formulation of aggregators taken from Epstein (1987) for continuous-time recursive representations. Specifically, under a set of plausible assumptions that can be found in Epstein (1987), there is no loss of generality by maintaining that the aggregator $f(c, J)$ can be written as

$$f(c, J) = U(c) - V(J), \tag{9}$$

where $U(\cdot)$ and $-V(\cdot)$ are concave.[4] Note that the aggregator as given in (9) is still (possibly) nonlinear in c and J.[5] The axiomatic foundations of the recursive utility in a continuous time deterministic setting parallels that of the discrete time settings of Epstein (1987) and Koopmans (1960). A central feature of the recursive functional form is that the rate of time preference is implicit in its structure. This leads to a flexible concept of discounting that depends systematically on an index of aggregate future opportunities, as captured by $J(t)$. With the Epstein (1987)'s aggregator (9), the dynamics of the social planner's valuation (7) becomes:

$$\frac{\dot{J}_X(t)}{J_X(t)} = R(K(t), X(t)), \tag{10}$$

---

[4] Examples of such function include the power utility; that is, $f(c, J) = \frac{c^\sigma}{\sigma} - V(J)$ with $\sigma < 1$.

[5] The time-additive utility corresponds to $f(c, J) = U(c) - rJ$, meaning that V is linear, i.e. $V(J) = rJ$.

where $R(\cdot,\cdot)$ is the function defined by $R(K(t),X(t)) = V^{'}(J(K(t),X(t))$. It is worth noting that $R(K(t),X(t))$ only depends on the state of the economy, as captured by the remaining stock size of the natural resource $X(t)$ and the stock of produced capital accumulated $K(t)$. Equation (10) is the basis of the pricing rule that we empirically assess in this paper.

3 Econometric Analysis

In this section, we perform an econometric analysis of the theoretical model using data on fossil fuels. We begin by describing the data and their salient features, and we specify the regressions model that we use to test the theoretical predictions. This is followed by a discussion of policy implications.

3.1 Data Description

One important issue in testing the theory of price evolution of nonrenewable natural resources has been the imperfect observability of the variables involved. We follow the literature by proxying the theoretical quantity $J_X(t)$ with the observed market price P_t (e.g. Schmidt 1988, Hamilton 2009). The stock of natural resources X_t is not directly available in the data, but the world quantity of proven oil reserves, S_t as well as the world quantity of extractions (proxied with oil production), x_t, are available. Likewise, the stock of capital used in the extraction technology is not available, so we take K_t as the total capital stock of OECD countries, which is strongly correlated with the desired capital.[6] The data on world oil prices, production and proven reserves come from the BP Statistics website (see also the OPEC statistics website and bulletins at http://stats.oecd.org/) and the data on capital stock is taken from the OECD website (see also The World Bank's website at http://data.worldbank.org/). The data are all annual and those on oil prices cover a period 1861-2013, whereas those on capital, reserves and extractions only cover the period 1960-2013. Most of our analysis is therefore performed on the latter period.

To build the history profile of the world's total stock of resource $\{X_t\}_{t=0}^{T}$, we take the most recent available evaluation of the world's oil proven reserves as X_T (which accounts for all the preceding discoveries to date), and we use the history profile of oil extractions

---

[6] The total stock of capital produced by OECD countries represents about 80% of the World's total. Both the stocks X_t and K_t used here are per-capita series.

$\{x_t\}_{t=0}^{T-1}$ to generate the remaining sequence as follows.[7]

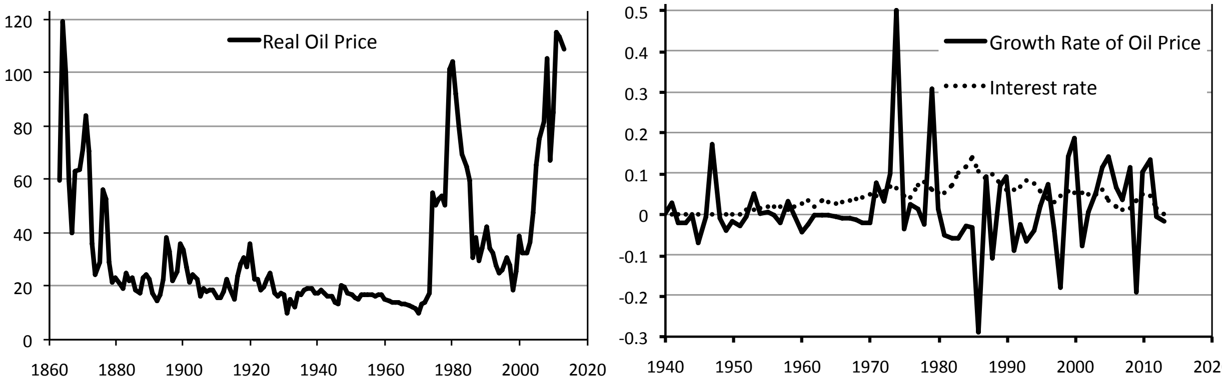
$$X_0 = X_T + \sum_{t=0}^{T-1} x_t, \quad \text{and} \quad X_t = X_{t-1} - x_{t-1}, \quad t = 1, \ldots, T$$

That is

$$\underbrace{X_t}_{Remaining\,stock\,at\,time\,t} = \underbrace{X_T}_{Today's\,proven\,reserves} + \underbrace{\sum_{\tau=t}^{T-1} x_\tau}_{Cumulative\,quantity\,extracted}, \qquad t = 0, \ldots, T-1 \quad (11)$$

In other words, we use the latest proven reserves X_T as a proxy for the remaining stock today. Our initial stock is therefore proxied as the sum of all the quantities discovered and extracted to date and the remaining stock.[8]

Figure 1: Real Oil Prices, Interest rates and Relative change in Prices (in 2013 Dollars)



The left panel of Figure 1 shows the evolution of oil prices over the period 1861-2013 (see also Hamilton 2009, Schmidt 1988) . In contrast to the theoretical predictions, real prices of petroleum have been highly volatile but have not exhibited a significant trend over the period 1861-2013. Current real prices are found at the levels of 100 years ago. In other worlds, oil prices have not exhibited the real appreciation that would be predicted by the basic Hotelling's rule. The right panel of Figure 1 shows a drastic discrepancy between

---

[7] According to the Securities and Exchange Commission (SEC), proved oil reserves are the quantities of oil, which, by analysis of geoscience and engineering data, can be estimated with reasonable certainty to be economically producible - from a given date forward, from known reservoirs, and under existing economic conditions, operating methods and government regulations.

[8] Our measure of resource stocks is obviously understated because the current proven reserves may not be an accurate measure of the true remaining stock. However, in the absence of any other credible method of evaluation, the most recent available information on oil proven reserves is the best measurement that we have for the remaining stock of oil in the ground.

relative price changes and interest rates, which further confirms that the former can not be explained by the latter as the Hotelling's rule would imply.

Figure 2: Capital stock (USD per capita) and Oil Stock (Barrels per capita)

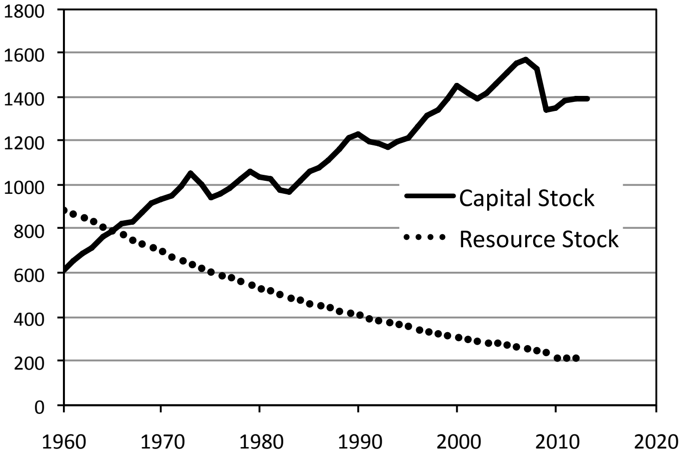


Figure 2 shows the evolution of the stock of capital and the stock of the resource per capita (world totals) over the period 1960-2013 (period where capital stock is available). The graphical analysis of these quantities provides a useful intuition about their possible impact on the evolution of prices. On the one hand, capital has increased over time, which would be associated with a decrease in the expected prices of the resources, since increased availability of capital lowers production costs. On the other hand, the decrease in the available stock of resources would be associated with an increase in the expected prices, since resource depletion creates scarcity rents. Together, these countervailing forces may help explain why the price of the resource has not sustainably appreciated or depreciated over time, as we argue in our theory. In what follows, we empirically assess the nature of the relationship between these quantities and the relative price changes.

3.2 Empirical Tests

We perform a set of parametric estimations and tests based on the Hotelling's rule and the linearized version of our pricing rule. Our main variables of interest are $p_t = \ln(P_{t+1}/P_t)$, the growth rate of prices, r_t the interest rate, K_t and X_t.[9] As is standard in time series analysis, we first performed stationarity tests on all our series using the Augmented Dickey-Fuller

---

[9] Or, equivalently, $p_t = (P_{t+1} - P_t)/P_t$. We actually also approximated the first derivative of the price using this difference score and the results were similar to the logarithm local approximation.

(ADF) and the Phillips-Perron (PP) unit root tests to assess their order of integration. As expected, p_t and r_t are stationary i.e. integrated of order 0, $I(0)$, whereas both K_t and X_t are difference-stationary, i.e. integrated of order 1, $I(1)$ (see Table 3). Basic tests of the Hotelling rule following directly from Equation (8) can be readily obtained. A simple evaluation of the relationship is to estimate the following equation

$$p_t = \alpha + \beta r_t + \epsilon_t \tag{12}$$

using ordinary least squares. The Hotelling model would then imply that $\alpha = 0$ and $\beta = 1$, and would be rejected otherwise. Since interest rate is stationary around a positive constant, another equivalent approach is to estimate the simple equation given by

$$p_t = \delta + \epsilon_t \tag{13}$$

The Hotelling model would then imply that $\delta > 0$, and would be rejected otherwise. The regression results for Equations (12) and (13) are presented in the first two colums of Table 2, where the data spans the period 1960-2013. These results show that the data do not support the Hotelling's rule. The coefficient β is negative and insignificant, and the joint test of the hypothesis that $\alpha = 0$ and $\beta = 1$ is rejected with an F-statistics of 5.9761 representing a p-value of 0.0046. Likewise, the coefficient δ is not significantly different from zero, with a t-test statistic of 0.95. In other words, the hypothesis $\delta = 0$ cannot be rejected in favor of the alternative hypothesis $\delta > 0$. Interestingly, while both models are inconsistent with the Hotelling's rule, the first is worse than the second since the Adjusted R-squared obtained from adding interest rate to the regression becomes negative. This means that the interest rate is irrelevant in explaining price growth and its inclusion further deteriorates the fit. That the basic Hotelling's model is not supported by the data is not new, and is not very surprising either. Intuitively, other factors such as the remaining stock of oil and the stock of capital useful for the extraction technology, must be accounted for in explaining price movements. Our pricing rule based on recursive preferences structurally includes such factors.

Under recursive preferences, Equation (10), which expresses the growth rate of prices p_t as a function of the stocks of capital and available resources through $R(K_t, X_t)$, forms the basis for the empirical evaluation of our pricing equation. Given that the value function is not observable, the functional form of $R(K_t, X_t)$ is unknown. But since it depends explicitly on the state variables K_t and X_t of the economy it can be linearly approximated for estimation purposes. We therefore start by assuming that $R(K_t, X_t)$ is a linear combination of the state variables K_t and X_t.[10] This corresponds to a baseline linear regression model defined by

$$p_t = \gamma_0 + \gamma_1 K_t + \gamma_2 X_t + \epsilon_t. \tag{14}$$

There are two practical issues surrounding the estimation of this model with time series. The first is that, unlike standard cointegration models where all variables included in the regression are $I(1)$, the dependent variable in this regression is $I(0)$ while the explanatory variables are integrated of order one, $I(1)$. So neither conventional statistical measures based on stationarity, nor cointegration statistics are applicable in this case. However, Pagan and Wickens (1989) explained that *"If the dependent variable is stationary, there must be either zero or at least two trending explanatory variables, one which is required to remove the growth of the other one and leave their combined explanation stationary"*. Our model fits exactly in this scenario where we have two explanatory variables K_t and X_t which are trending in opposite directions. Baffes (1997) therefore proposed a formal test that consists in estimating the model by ordinary least squares (OLS), test the stationarity of the residuals and the joint significance of the two $I(1)$ regressors. These tests are performed and reported in Table 2 under the column 'OLS', and they show that both the capital and resource coefficients are significantly positive and the residuals are stationary at a 1% level of significance.

The second issue is the possible endogeneity of current stock of capital and oil stock, due to the possible reversed causality of these factors with crude oil prices at time t. The usual solution is to use instrumental variables which include lagged values of these explanatory variables. We also include in the set of instruments the number of U.S. crude oil exploratory and developmental wells drilled per capita (denoted W_t). As shown in Table 3, this variable is also $I(1)$ and is expected to be possibly correlated to changes in oil prices at time t, but only through capital and/or oil stocks. Using these instruments, the estimation proceeds in two-stage least squares where the first stage is the capital stock and oil stock determination equations and the second stage is the price growth determination equation. Table 1 reports the first stage regression results of capital and oil stocks determination equations. The results indicate that both equations exhibit satisfactory performance in terms of expected signs and $F-$ statistics (capturing the strenght of the instruments). The last two rows of Table

---

[10] In the absence of further information, the linear approximation allows to get at least an intuitive sense of the amount of correlations that is contained in the equilibrium relationship. It is important to note that this is not a linear approximation of the value function, but only that of the derivative function $V'(J(K, X))$ featured at the right hand side of Equation (10)

Table 1: First stage regression

	K_t	X_t
Intercept	0.4060***	0.0674
	(0.1222)	(0.0771)
K_{t-1}	0.7158***	-0.1327 **
	(0.0895)	(0.0564)
X_{t-1}	-0.0189**	0.9702***
	(0.0076)	(0.0047)
W_t	-0.8250***	-0.1460*
	(0.2694)	(0.0898)
R^2	0.9811	0.9999
F-stat	656.36	600.39
$ADF^\dagger$	-4.273 ***	-4.929 ***
$PP^\dagger$	-4.162***	-4.434***

Standard errors in parenthesis. Significance: "***" 1%, '**" 5%, '*" 10%

†Cointegration statistics. Critical values: -3.634 (1%), -2.952 (5%), -2.610 (10%).

1 report cointegration statistics (ADF and PP). All statistics indicate that the disturbance term of both equations is stationary at the 1% level of significance, hence establishing the validity of the instruments. Table 3 also reports tests regarding the order of integration of the predicted values of capital and oil stocks denoted $\widehat{K}_t$ and $\widehat{X}_t$ and confirm that they are both $I(1)$, just like K_t and X_t. In the second stage, the pricing equation is estimated as a function of the real capital and oil stocks per capita predicted from the first stage (instrumental variable estimation). The results are reported in Table 2 under the column denoted 'IV'. Both the capital stock and oil reserve coefficients are positive as expected from theory. Moreover, the regression results provide overwhelming evidence of a stationary disturbance, a result supported by the stationarity tests at a 1% level of significance, as recommended by Baffes (1997). Based on these findings, we conclude that the model performs in a satisfactory manner and therefore interpret the elasticities and the goodness of fit statistics accordingly.

The estimation results from both OLS and IV show that none of them is rejected by the data. The F-test statistic shows that the models are globally significant. All the coefficients are statistically significant at the 5% level, but the OLS model seem to have a better explanatory power than the IV model, as implied by the corresponding R-squared values. There are several ways to compare the recursive preferences based (that we refer to as RPB) pricing model to the Hotelling's model. First, the adjusted R-squared in the OLS and IV

Table 2: Empirical Test of the Pricing Rules

	Hotelling rule		RPB Pricing Rule	
	(1)	(2)	OLS	IV
Intercept	0.0151	0.0419	-0.9396**	-1.0289**
	(0.0161)	(0.0340)	(0.3998)	(0.4471)
Interest rate		-0.5214		
		(0.5850)		
Capital			0.7195**	0.7857**
			(0.2999)	(0.3352)
Resource			0.0688**	0.0756**
			(0.0288)	(0.0318)
R^2	0.0000	0.0150	0.1543	0.1245
R^2_{Adj}	0.0000	-0.0039	0.1211	0.1078
F-stat	0.0000	0.7943	3.932**	3.843**
$ADF^\dagger$			-6.474 ***	-6.756 ***
$PP^\dagger$			-6.475 ***	-6.761***

Notes. Standard errors in parenthesis. Significance codes: "***" 1%, '**' 5%, '*' 10%.
$^\dagger ADF$ and PP stationarity tests for the residuals. The critical values are -3.634 (1%), -2.952 (5%), and -2.610 (10%).

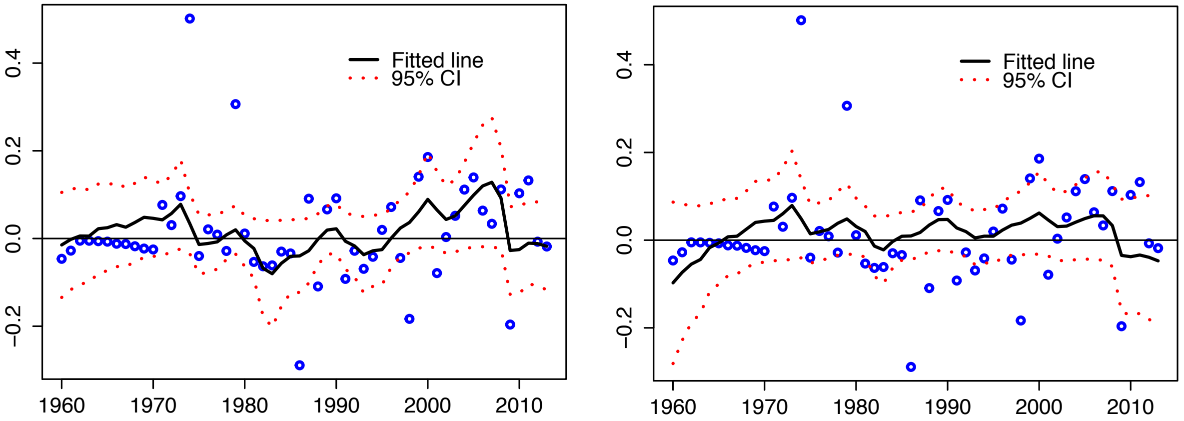
regressions are significantly higher than those of the Hotelling models. This means that there is an important increase in fit when the set of regressors include the stocks of capital and natural resources, compared to the regression on the constant and/or interest rate. In fact, the explanatory power of the OLS is more than 10 times higher than that of the Hotelling's model. Second, the joint test of overall significance in OLS and IV (given by the F-tests in the table) can be seen as a direct model selection test between the RPB pricing rule and the Hotelling's rule. It implicitly tests a null hypothesis given by Equation (13) against the hypothesis that the stocks of capital and available resources are jointly relevant predictors of the price growth rate, i.e. Equation (14). The associated F-tests are computed at 3.93 and 3.84, respectively for the OLS and IV regressions, thus preferring the RPB pricing rule over the Hotelling's rule at a the 5% statistical significance. Figure 3 depicts the actual and fitted values of the price growth rate for both the OLS and IV regressions. Overall, the estimation shows a fair proximity between the two values. The fitted model, however, does not pick up extreme values of the depended variables since linear regression puts higher weights on values near sample averages. This is well illustrated when the 95% confidence band is added to the figures and is expected to cover most of the original data points. Interestingly, like the actual values, the predicted values are mean-reverting and cover a range of both negative

Table 3: Stationarity (unit-root)Tests

		Levels						First difference				
	p_t	r_t	K_t	X_t	W_t	$\widehat{K}_t$	$\widehat{X}_t$	ΔK_t	ΔX_t	ΔW_t	$\Delta \widehat{K}_t$	$\Delta \widehat{X}_t$
ADF	0.000	0.015	0.438	0.477	0.446	0.599	0.566	0.000	0.000	0.000	0.003	0.002
PP	0.000	0.016	0.646	0.651	0.335	0.641	0.650	0.002	0.001	0.000	0.002	0.000

Notes. P-values for testing the null hypothesis of a unit root using ADF and PP tests are reported.
These results show that p_t and r_t are $I(0)$, while $K_t, X_t, W_t, \widehat{K}_t$,and $\widehat{X}_t$) are $I(1)$, at conventional levels.

Figure 3: Parametric Estimation: OLS (left) and IV (right)



and positive changes in oil prices.

3.3 Discussion and Implications

Recursive preferences provide a rich structure to explore attitudes over the states of the economy through a state dependent (endogenous) discount factor. As shown by the empirical results, there are two forces working in opposite directions crucially shaping the implied endogenous discount factor over time. On the one hand, decreases in oil stock tend to reduce the endogenous discount factor, therefore expressing impatience and less concern for the future. On the other hand, increases in the size of the produced capital tend to increase the discount factor, expressing the desire to build up future-oriented capital.

Our findings may also have interesting implications for understanding the evolution of crude oil prices whose recent plunge has renewed interest in the debate of what really determines their spot and future values. The above results confirm that contrary to the Hotelling's rule, the interest rate is not necessarily a direct fundamental determinant of oil prices. Our analysis suggests that the predicted next period oil price is the current spot price adjusted by a factor that should vary with both the resource stock and the stock of man-made capital in the production technology. In other words, since $p_t = \ln(P_{t+1}/P_t)$ and by the preceeding analysis $\hat{p}_t = \widehat{R}(K_t, X_t)$, it follows that the predicted price of crude oil is

$$\widehat{P}_{t+1} = P_t \exp\{\widehat{R}(K_t, X_t)\}, \tag{15}$$

where $\widehat{R}(K_t, X_t)$ is the estimated regression function as described above.

This equation provides some insights for the price analysis of the crude oil market. The approach used in the literature to predict oil prices are usually based on interest rates, current spot prices, and/or futures spreads, e.g. Alquist and Kilian (2010). However, our findings suggest that oil and capital stocks are structural statistically significant drivers of oil prices with substantial explanatory power and should therefore also be included in the price determination equations. While for oil, the unpredictability and disturbances are likely to be influenced by the possibility of geopolitical tensions, sudden changes in OPEC's policy or objectives, demand shocks, and appreciation/depreciation of the U.S. dollar, in the long run, it is expected that the depletion of oil stock should exert a downward pressure on real oil prices while the increase in produced capital should exert an upward pressure on these prices (through their countervailing effects on the endogenous discount factor).

In terms of economic policy, since changes in oil prices would significantly affect inflation rate and more specifically local food prices as recently found by Dillon and Barrett (2016), policymakers concerned about the impacts of oil prices on poverty and food security should account for the aforementioned factors and related shocks. Our findings suggest that structural policies would differ based on whether a country is an oil producer or importer, and on their extent of technological progress. This is particularly relevant to investors in capital markets interested in gaining a better understanding of the structural factors that could help determine the value of oil assets while making their strategic portfolio decisions (Kakeu and Bouaddi, 2016).

4 Conclusion

The purpose of this paper is to derive and test a structural equilibrium pricing rule for crude oil using a recursive preferences approach in a capital-resource economy model. The significance of this study is that it tests an alternative theoretical model that could help explain the long-run evolution of crude oil prices in nonrenewable resource markets from a theoretical perspective. We show that under recursive preferences, a pricing rule that predicts the relative change in prices as a function of the capital stock and the available resource will obtain. We thus posit the depletion of the resource and the increasing availability of capital as two countervailing forces on prices that may help explain why crude oil prices have not appreciated in real terms over the past century despite important fluctuations as found in many studies. To assess the empirical relevance of this theory, we perform a set of parametric estimations and tests using data from oil prices, world capital stock, world oil production and proven reserves. The results based on both OLS and IV regressions provide compelling evidence that the pricing rule derived from a recursive preference formulation is consistent with the data. Our findings provide insights for the long term determination of crude oil prices and the design of related energy policy responses. Since the dynamic behaviour of resource prices discussed in this paper is derived from a structural approach, this framework may provide a suitable basis for structural estimation and counterfactual simulations useful for evidence-based policy prescription. This is left for future research.

References

Agbeyegbe, T. (1989) 'Interest rates and metal price movements: Further evidence.' *Journal of Environmental Economics and Management* 16(2), 184–192

Alquist, R., and L. Kilian (2010) 'What do we learn from the price of crude oil futures?' *Journal of Applied Econometrics* 25, 539–573

Baffes, John (1997) 'Explaining stationary variables with nonstationary regressors.' *Applied Economics Letters* 4(1), 69–75

Boyd III, J. (1990) 'Recursive utility and the ramsey problem.' *Journal of Economic Theory* 50(2), 326 – 345

Browning, M. (1991) 'A simple nonadditive preference structure for models of household behavior over time.' *Journal of Political Economy* 99(3), 607–637

Dasgupta, P., and G. Heal (1974) 'The optimal depletion of exhaustible resources.' *The Review of Economic Studies* 41, 3–28

Dillon, B. M., and C. B. Barrett (2016) 'Global oil prices and local food prices: Evidence from east africa.' *American Journal of Agricultural Economics* 98, 154–171

Duffie, D., and L. Epstein (1992) ‘Asset pricing with stochastic differential utility.’ *Review of Financial Studies* 5, 411–436

Epstein, L. (1987) ‘The global stability of efficient intertemporal allocations.’ *Econometrica* 2, 329–355

Epstein, L., and J. Allan (1983) ‘The rate of time preference and dynamic analysis.’ *Journal of Political Economy* 91, 611–35

Frederick, S., G. Loewenstein, and T. O’Donoghue (2002) ‘Time discounting and time preference: A critical review.’ *Journal of Economic Literature* 40, 351–401

Gaudet, G. (2007) ‘Natural Resource Economics under the Rule of Hotelling.’ *Canadian Journal of Economics* 40, 1033–1059

Geoffrey, H., and M. Barrow (1981) ‘Empirical investigation of the long-term movement of resource prices: A preliminary report.’ *Economics Letters* 7, 95 – 103

Gorman, W. (1968) ‘The structure of utility functions.’ *Review of Economic Studies* 35, 367–390

Halvorsen, R., and T.R. Smith (1991) ‘A test of the theory of exhaustible resources.’ *The Quarterly Journal of Economics* 106, 123–140

Hamilton, James D. (2009) ‘Understanding crude oil prices.’ *The Energy Journal* 30, 179–206

Heal, G., and M. Barrow (1980) ‘The relationship between interest rates and metal price movements.’ *Review of Economic Studies* 47(1), 161–81

Hertzendorf, M. (1995) ‘Recursive utility and the rate of impatience.’ *Journal Economic Theory* 5, 51–65

Howitt, R. E., S. Msangi, A. Reynaud, and K. Knapp (2005) ‘Estimating intertemporal preferences for natural resource allocation.’ *American Journal of Agricultural Economics* 87, 969–983

Kakeu, J., and M. Bouaddi (2016) ‘Empirical evidence of news about future prospects in the risk-pricing of oil assets.’ *Energy Economics*

Knapp, K. C., and L. J. Olson (1996) ‘Dynamic resource management: Intertemporal substitution and risk aversion.’ *American Journal of Agricultural Economics* 78, 1004–1014

Koopmans, T. (1960) 'Stationary ordinal utility and impatience.' *Econometrica* 28(2), 287–309

_ (1986) *Representation of Preference Ordering with Independent Components of Consumption* (Minneapolis: University of Minnesota Press)

Krautkraemer, J. (1998) 'Nonrenewable resource scarcity.' *Journal of Economic Literature* 36, 2065–2107

Livernois, J. (2009) 'On the Empirical Significance of the Hotelling Rule.' *Review of Environmental Economics and Policy* 3, 22–41

Livernois, J., and P. Martin (2001) 'Price, scarcity rent, and a modified r per cent rule for non-renewable resources.' *Canadian Journal of Economics* 34, 827–845

Livernois, J., H. Thille, and X. Zhang (2006) 'A test of the hotelling rule using old-growth timber data.' *Canadian Journal of Economics* 39(152), 543–559

Mak, K. (1986) 'On Separability: Functional Structure.' *Journal of Economic Theory* 40(2), 250–282

Pagan, A. R., and M. R. Wickens (1989) 'A survey of some recent econometric methods.' *The Economic Journal* 99(398), 962–1025

Peltola, J., and K. Knapp (2001) 'Recursive preferences in forest management.' *Forest Science* 47, 455–465

Samuelson, P. (1937) 'A note on measurement of utility.' *The Review of Economic Studies* 4(2), pp. 155–161

Schmidt, R. (1988) 'Hotelling's rule repealed? an examination of exhaustible resource pricing.' *Economic Review-Federal Reserve Bank of San Francisco* (Fall 1988), 41–54

Skiadas, C. (2007) 'Chapter 19 dynamic portfolio choice and risk aversion.' vol. 15 pp. 789–843

Slade, M. (1982) 'Trends in natural resource commodity prices: An analysis of the time domain.' *Journal of Environmental Economics and Management* 9, 122–137

Slade, M., and H. Thille (1997) 'Hotelling Confronts CAPM: A Test of the Theory of Exhaustible Resources.' *Canadian Journal of Economics* 30, 685–708

Solow, R. (1974) ‘Intergenerational equity and exhaustible resources.’ *Review of Economic Studies* 41, 29–45

Stiglitz, J. (1974) ‘Growth with exhaustible natural resources: Efficient and optimal growth paths.’ *The Review of Economic Studies* 41, pp. 123–137

Streufert, P. (1995) ‘A general theory of separability for preferences defined on a countably infinite product space.’ *Journal of Mathematical Economics* 24(5), 407–434

Young, D. (1992) ‘Cost specification and firm behaviour in a Hotelling model of resource extraction.’ *Canadian Journal of Economics* 25, 41–59

A Appendix: Deriving the evolution of the resource shadow price with a recursive utility

Denote by $J(K(t), X(t))$ the value function, maximized utility, of the representative consumer in state $(K(t), X(t))$ at time t. Then the corresponding Hamilton-Jacobi-Bellman of the planner's problem (3)-(4)-(5)-(6) is given by:

$$0 = \sup_{c,x} \left[f(c(t), J(K(t), X(t))) + \dot{X}(t)J_X(t) + \dot{K}(t)J_K(t)\right], \tag{16}$$

subject to:

$$\begin{aligned} \dot{X}(t) &= -x(t), \qquad (17) \\ \dot{K}(t) &= F(K(t), x(t)) - c(t) - \delta K(t). \qquad (18) \end{aligned}$$

At time t, the corresponding Hamilton-Jacobi-Bellman of the planner's problem (3)-(4)-(5)-(6) is given by:

$$0 = \sup_{c,x} \left[f(c(t), J(K(t), X(t))) + \dot{X}(t)J_X(t) + \dot{K}(t)J_K(t)\right], \tag{19}$$

subject to:

$$\begin{aligned} \dot{X}(t) &= -x(t), \qquad (20) \\ \dot{K}(t) &= F(K(t), x(t)) - c(t) - \delta K(t). \qquad (21) \end{aligned}$$

In equilibrium, the maximized Bellman equation corresponding to the problem (19)-(20)-(21) is given by

$$f(c(t), J(K(t), X(t))) - x(t) J_X(K(t), X(t)) + [F(K(t), x(t)) - c(t) - \delta K(t)] J_K(K(t), X(t)) = 0. \tag{22}$$

Differentiating the maximized Bellman equation (22) with respect to X and using the envelope theorem gives

$$f_J(t) J_X(t) - x(t) J_{XX}(t) + [F(K(t), x(t)) - c(t) - \delta K(t)] J_{KX}(t) \;=\; 0 \tag{23}$$

which becomes

$$f_J(t) J_X(t) + \dot{J}_X(t) \;=\; 0 \tag{24}$$

where the time derivative of the resource shadow price $\dot{J}_X(t) = \frac{dJ_X(K(t),X(t))}{dt} = -x(t) J_{XX}(t) + [F(K(t), x(t)) - c(t) - \delta K(t)] J_{KX}(t)$.[11]

From Equation (24) it follows that the dynamics of shadow prices is given by:

$$\frac{\dot{J}_X(t)}{J_X(t)} \;=\; -f_J(c(t); J(K(t), X(t))). \tag{25}$$

---

[11] For ease of notation, throughout we shortly use $J(t)$ to refer to $J(K(t), X(t))$, $f(t)$ to refer to $f(x(t), J(t))$, $F(t)$ to refer to $F(K(t), x(t))$, unless otherwise stated.