Does More Recycling Always Mean More Trees?

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Abstract

Interest in paper recycling has grown significantly, driven by the goal of preserving forest stocks and their associated carbon sequestration benefits. This paper presents a dynamic equilibrium model to examine the relationship between the paper manufacturing industry, which utilizes recycled fiber, and the logging industry, which supplies virgin wood. We theoretically derive the socially optimal recycling path and demonstrate that intensive recycling can inadvertently disincentivize forest carbon sequestration by reducing the marginal return on timberland investment, leading to land-use conversion. Using USDA Forest Service data (1965-2015) for structural estimation, we find that the socially optimal recovery rate converges to a steady state of approximately 58% while actual recovery rates have trended higher. This suggests that current policy intensity may backfire by suppressing timber prices below the level necessary to sustain private investment in forest land. We conclude that achieving an efficient circular economy requires recycling targets that account for the price-induced displacement of primary resource investments.

Keywords: Optimal control, Recycling, Paper, Forestry, Land allocation.

JEL codes: Q23, Q24, C13.

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1 Introduction

Over the last four decades, rising environmental awareness and the climate crisis have catalyzed global policy efforts to mitigate carbon emissions and ecosystem degradation. A cornerstone of these efforts has been the aggressive promotion of paper recycling through public campaigns, municipal subsidies, and fiscal incentive measures recently reaffirmed in the EPA's 2020 Recycling Economic Information (REI) Report. The prevailing logic underpinning these policies is that secondary fiber recovery serves as a direct proxy for forest conservation; by reducing the demand for virgin wood, recycling is assumed to preserve the standing timber stocks that provide critical ecosystem services, including carbon sequestration and soil stabilization. Consequently, the global recovery of paper and paperboard has reached unprecedented levels. In the United States, for instance, the recovery rate surged from 22.8% in 1965 to 66% in 2015, a nearly three-fold increase in the volume of diverted material (American Forest and Paper Association, 2016). However, this conventional wisdom overlooks the endogenous response of land-use markets to shifts in relative commodity prices.

However, the pursuit of exhaustive recycling targets may carry overlooked risks for long-term resource sustainability. Given that a substantial portion of timberland is privately held, the expansion of secondary fiber markets directly impacts the profit margins of the logging industry and, by extension, the incentives for reforestation. While recent scholarship has highlighted operational inefficiencies — such as fiber degradation (Lafforgue and Rouge, 2019; Lafforgue and Lorang, 2022), escalating processing costs (Yamamoto and Kinnaman, 2022), and geopolitical shifts in scrap markets (Hook and Reed, 2018) — the fundamental economic tension lies in the relative return on land. In the United States, empirical trends reveal a stark divergence: while the paper recovery rate surged after 1989, the area of annual forest planting entered a period of continuous decline, falling by 29.1% by 2015 (USDA). This inverse correlation suggests a potential market displacement effect, where high recycling rates depress virgin wood prices to the point of creating a structural disincentive for timber production. This study investigates this relationship by developing a dynamic general equilibrium model to identify the socially optimal recycling threshold and evaluate whether current U.S. policy intensity has moved the market into a regime of diminishing environmental returns.

We formalize these dynamics through a dynamic land-allocation model where the equilibrium stocks of virgin timber and forested acreage are jointly determined. The model captures the structural interdependence between the paper manufacturing sector, which demands virgin fiber as a primary input, and the logging industry, which optimizes land-use between forestry and alternative productive uses. We first characterize the socially optimal trajectory by solving a social planner's problem that internalizes non-market externalities, including forest-based amenity benefits, the marginal cost of recovery, and the social disutility of waste. This theoretical framework reveals a non-monotonic relationship between recycling rates and forest inventory, identifying a critical threshold beyond which marginal increases in recycling become ecologically counterproductive. Second, we derive the decentralized general equilibrium path, allowing us to quantify the wedge between market-driven recovery rates and the socially optimal state. By estimating the model's structural parameters using U.S. data where private ownership dominates the timber landscape, we quantify the divergence between historical recycling trends and the derived optimal path. While factors such as urban expansion and agricultural shifts contribute to national land-use trends, our structural approach allows us to isolate the price-transmission mechanism specifically linked to the secondary fiber market. By identifying the elasticity of land supply relative to timber prices, we control for exogenous land-value shocks, ensuring that the identified 58% optimum reflects the inherent trade-offs between fiber recovery and forest-stand maintenance rather than broader macroeconomic shifts in real estate.

The model developed herein provides a theoretical formalization and expansion of the "price-response" intuition first posited by Darby (1974). Darby argued that for private landowners, recycling mandates could inadvertently reduce long-run forest inventory by depressing timber prices, an observation that remained largely unexplored for decades due to data limitations and nascent recycling markets. While subsequent studies, such as Wiseman (1993) and Ince (1995), utilized input-output frameworks to hypothesize a qualitative decline in virgin wood demand, they stopped short of modeling the endogenous land-allocation dynamics or quantifying the magnitude of the impact. Later theoretical work, including Tatoutchoup and Gaudet (2010), integrated the Faustmann (1849) infinite-rotation framework to formalize these land-use trade-offs, and Kinnaman et al. (2014) estimated social cost-minimizing recycling rates in Japan, albeit without accounting for upstream timber production or land competition. This paper distinguishes itself from the existing literature, particularly Tatoutchoup (2016), through three primary contributions. First, rather than focusing solely on the social plannerâs optimum, we explicitly model the decentralized market equilibrium. This allows for a rigorous assessment of the welfare "wedge" created by current policy incentives. Second, we move beyond purely theoretical analysis by employing structural estimation on fifty years of USDA Forest Service data. By defining specific functional forms for land-use elasticity and timber demand, we provide the first empirical test of the price-induced forest displacement hypothesis. Finally, our framework enables counterfactual simulations to evaluate the efficiency of alternative policy instruments, shifting the focus from volume-based recycling targets toward optimal social welfare.

While our analysis centers on the price-depressing effects of secondary fiber recovery, we acknowledge that U.S. forest dynamics are influenced by broader structural transitions. As noted by Berck (1979) and Brazee and Mendelsohn (1990), the industry has undergone a fundamental shift from old-growth harvesting to a renewable, age-delimited stock management paradigm. Furthermore, Sedjo and Lyon (1990) identify a shift from an 'extensification frontier' (adding new hectares) to an 'intensification frontier' (increasing management inputs per hectare), suggesting that a slowdown in total acreage may partially reflect higher productivity per acre. Most recently, Mendelsohn and Sohngen (2019) observe that a deceleration in timber price growth and, consequently, planting can be attributed to the maturation of this transition and increased forest investments. By integrating these perspectives, our study isolates the marginal impact of recycling within this evolving landscape, identifying it as a distinct factor that exacerbates the price deceleration and further disincentivizes land extensification.

The paper is organized as follows. Section 2 presents the theoretical framework, determines the optimal social outcome, and identifies the optimal recycling path. In Section 3, we first set up the private-market equilibrium conditions required to estimate model parameters. We provide structural estimates of agents'utility, firms' production, and external benefits and costs. After that, we quantify the optimal social outcome. Finally, Section 4 concludes the paper and discusses some policy options.

2 Analytical Framework

The model developed here theoretically formalizes and expands an argument that was initially posited by Darby (1974). We mainly refer to a country as the scale of the analysis, but it could also be understood or extended to a state, region, or county where environmental policies are well circumvented. Keeping in mind the data constraints, we follow the notation and context of Tatoutchoup and Gaudet (2010) and Tatoutchoup (2016), assuming that at each time t, the quantity of paper, $q(t)$, which is a recyclable good, is produced using virgin wood $v(t)$ and recycled inputs $w(t)$, that is, paper waste recovered in the previous period. The paper waste recovered is the portion of the country's total consumption waste that was recovered. The total consumption of the country $c(t) = q(t) + \bar{x}$ at each time t is the sum of the domestic production of paper $q(t)$ and the net import ($\bar{x}$), which we assume to be constant. This net import is the total import of paper to the country minus its total exports. If we denote by $\delta(t)$, with $0 \leq \delta(t) \leq 1$, the recycling rate defined as the proportion of paper from the previous period recovered by households after consumption, then we have $w(t) = \delta(t)(q(t) + \bar{x})$. The paper manufacturing industry, which we assume to be competitive, produces the final output, paper, through the production function $G(v(t), w(t))$. The function $G(v, w)$ is continuously differentiable, strictly increasing in its two arguments (i.e., $G_v(v, w) > 0$, $G_w(v, w) > 0$), homogeneous of degree one, quasi-concave, and satisfies $G(0, 0) = 0$.[1] The fact that $G(v, w)$ is quasi-concave and homogeneous of degree 1 implies that $G_{vv}(v, w) \leq 0$, $G_{ww}(v, w) \leq 0$, and $G_{vw}(v, w) \geq 0$. Finally, the rate of change in the final output $q(t)$ is given by:[2]

$$\dot{q}(t) = G(v(t), \delta(t)(q(t) + \bar{x})) - q(t) \tag{1}$$

where a dot denotes the derivative with respect to time.

The logging industry, which we assume to be competitive, supplies the virgin input $v(t)$ through forest management activities, including the planting and harvesting of trees, while using land as an input. We assume that there is another competing use of land, say agriculture, such that the logging industry allocates a fixed land area, A, each time t between these two competing uses. Thus, at each time t, $F(t)+a(t) = A$, where $F(t)$ denotes the total area of forests used for production (including planted and naturally regenerating forests) and $a(t)$ denotes the total area of land devoted to agriculture. We assume that at time t, the total area of forests used for production is a multiple of the current planting area denoted by $f(t)$. Hence, $F(t) = sf(t)$ where the multiplier $s > 1$ is assumed to be exogenously fixed over time for simplicity. Therefore, the allocation of land can be rewritten as:[3]

$$sf(t) + a(t) = A \quad \text{for all } t \geq 0 \tag{2}$$

[1]For any given function $G(v, w)$, we denote by $G_v(v, w) = \partial G(v, w)/\partial v$, $G_w(v, w) = \partial G(v, w)/\partial w$, and $G_{vw}(v, w) = \partial^2 G(v, w)/\partial v \partial w$.

[2]In discrete time, the production function at time t is $q(t+1) = G(v(t), \delta(t)(q(t) + \bar{x}))$, so that $q(t+1) - q(t) = G(v(t), \delta(t)(q(t) + \bar{x})) - q(t)$.

[3]This multiplier is a stationary time series around an average value $s > 1$ in the data.

To model the problem of the logging industry for a given land area, the economic analysis of forest management is based heavily on determining the optimal age of trees, known as the optimal rotation age. This concept goes back to Faustmann (1849) for a homogeneous forest. Subsequent works, such as Mitra and Wan (1985, 1986) and Salo and Tahvonen (2003, 2004), extend the analysis to age-structured forests, where the forest is represented as a set of age classes, each associated with a land area in the discrete case. Recently, Fabbri et al. (2015) explored this problem in continuous time, which provides a better representation of the forest resource. However, all of these studies require knowledge of the species-specific growth functions of the harvested trees to determine the optimal age of the trees, making it difficult to adopt this approach for our empirical exercise. In light of these challenges and drawing from earlier research such as Barbier and Burgess (1997) and Lopez, Shah, and Altobello (1994), we assume that the forest area is the single input used to produce the forest product, denoted $h(t)$.[4]

Since the total forest used for production, $F(t)$, is proportional to the planting area $f(t)$, we then assume that $h(t) = H(f(t))$, where the harvest function $H(f)$ is increasing, twice differentiable, and concave, that is $H_f(f) \geq 0$, and $H_{ff}(f) \leq 0$. Hence, the quantity of input in the production of paper, $v(t)$, can be written as

$$v(t) = H(f(t)) - y(t). \tag{3}$$

where $y(t)$ is a part of virgin timber used in another sector of activities, such as construction, $y(t)$ is assumed to be exogenous.

The final output of other competing land uses (i.e., agriculture), $z(t)$, is produced using the land area as a single input. Hence, $z(t) = B^a(a(t))$, where the function $B^a(a)$ is assumed to be twice differentiable and concave (i.e., $B^a_{aa}(a) \leq 0$). Finally, the forest generates

[4]This assumption facilitates the analytical tractability and empirical assessment of the model, which is a strength, while also overlooking the potential real-life complexity of forestry, which is a weakness. As discussed later, our empirical results based on this assumption are encouraging, as variation in forest area alone can explain about 70% of the variability in the volume of wood harvested in our data.

ecosystem services (e.g., wildlife habitat, carbon sequestration, protection against soil erosion, etc.) that are valued by society (but potentially not captured by the market) and given by $B^e(f(t))$.[5] We assume that $B^e(f)$ is twice differentiable and concave (i.e. $B^e_{ff}(f) \leq 0$).

It is worth noting that the concavity of the harvest function $H(\cdot)$ can be interpreted in terms of decreasing land quality; that is, as the planting area increases, forestry expands to lower quality areas of poorer productivity and, hence, output. It could also be interpreted as diminishing returns to scale, e.g., due to limitations in managerial capacity, as is usually assumed in the literature (e.g., Samuelson 1976, Poore et al. 1989, Barbier et al. 1997). These interpretations similarly hold for the functions $B^a(\cdot)$ and $B^e(\cdot)$. The fact that both the wood and agricultural production functions increase with their respective land allocations implies that, at the margin, the trade-off is between the relatively more productive use, given that the same land pool is divided between forestry and agriculture.

2.1 Social Optimum

We first examine the optimal solution when the social planner considers the forest's positive externalities in terms of the amenity benefits generated by the forest, the social recycling cost, the societal cost of not recycling incurred by the community, and consumer preferences. Households are assumed to derive their utility at each time t through the consumption of paper $c(t)$ and the competing use of land output $z(t)$. The preferences of the representative consumer at time t, $U(c(t), z(t)) = u(c(t)) + z(t)$, are assumed to be quasilinear in $z(t)$. The utility function $u(c(t))$ is assumed to be twice differentiable and strictly concave ($u''(c) = u_{cc}(c) < 0$).

Finally, after consumption, the fraction of the final product that is not recycled, $c(t) - w(t) = (1 - \delta(t))(q(t) + \bar{x})$, will be disposed of as waste. Both recycling and landfill services

[5] Note that while we assume that forests provide several ecosystem services in addition to timber production in general, there are instances where some of these forests (e.g., industrial mono-culture plantations) may be of notably lower value in terms of wildlife habitat compared to other land uses such as agriculture. We thank an anonymous referee for pointing this out.

impose a cost on society. Recycling costs include collecting, separating, processing, and transporting recyclable paper. However, landfill costs include waste-disposal charges and environmental costs (i.e., additional pollution resulting from increased landfill use). If we denote by $\theta(t)$ the marginal cost of the landfill at time t, which is also the marginal cost of failing to recycle, then the total social cost of not recycling is $\theta(t)(1-\delta(t))(q(t)+\bar{x})$. Likewise, we denote by $C(w(t))$ the total social cost of recycling and assume that it is increasing, twice differentiable, and convex (i.e. $C_w(w) > 0$ and $C_{ww}(w) \geq 0$).

The social welfare, denoted W, is then defined as the sum of the discounted representative consumer's utility, $U(c(t), z(t))$. The ecosystem service benefit $B^e(f(t))$, net of both the recycling $C(\delta(t)(q(t) + \bar{x}))$ and the cost of failing to recycle $\theta(1 - \delta(t))(q(t) + \bar{x})$. Therefore, the problem of the social planner is:[6]

$$W = \max_{q,\ f,\ \delta} \int_{t_0}^{+\infty} [U(q + \bar{x}, B^a(A - sf) + B^e(f) - C(\delta(q + \bar{x})) - \theta(1 - \delta(q + \bar{x})]e^{-rt}dt \quad (4)$$

subject to

$$\dot{q} = G(H(f) - y, \delta(q + \bar{x})) - q \quad (5)$$

$$0 \leq \delta \leq 1 \quad (6)$$

$$q \geq 0,\ \ f \geq 0 \quad (7)$$

$$q(t_0) = q_0. \quad (8)$$

The social planner problem (4)-(8) is an optimal control problem, where q is the state variable and f and δ are the control variables. We substitute $a = A - sf$ from Equation (2) into $B^a(a(t))$ and $z = B^a(a)$ into the objective function (4). In addition, the constraint (5) follows by substituting $v = H(f) - y$ from Equation (3). The parameter r is the discount rate. The current-value Hamiltonian associated with the control problem above is written

$$\begin{aligned} \mathcal{H}(q, f, \delta) \quad &= \quad U(q + \bar{x}, B^a(A - sf)) + B^e(f) - C(\delta(q + \bar{x})) - \theta(1 - \delta)(q + \bar{x}) \\ &\quad +\mu(G(H(f) - y, \delta(q + \bar{x})) - q) \end{aligned} \quad (9)$$

[6]To simplify notations, time subscripts have been omitted where there is no risk of confusion between variables and constants.

The variable μ is the time-dependent current costate variable associated with the equation of motion (5). The necessary first-order conditions for an optimal trajectory are given by Equations (10) -(13), and the Appendix includes the sufficient conditions for optimality.

$$r\mu - \dot{\mu} = u'(c) - \mu - \delta(C_w(\delta(q+\bar{x})) - \mu G_w(v, \delta(q+\bar{x}))) - (1-\delta)\theta \tag{10}$$

$$\mu G_v(v, \delta(q+\bar{x}))H_f(f) + B_f^e(f) = sB_a^a(A - sf) \tag{11}$$

$$\delta = 0 \text{ only if } \quad C_w(\delta(q+\bar{x})) - \mu G_w(v, \delta(q+\bar{x})) > \theta \tag{12a}$$

$$0 < \delta < 1 \text{ only if } \quad C_w(\delta(q+\bar{x})) - \mu G_w(v, \delta(q+\bar{x})) = \theta \tag{12b}$$

$$\delta = 1 \text{ only if } \quad C_w(\delta(q+\bar{x})) - \mu G_w(v, \delta(q+\bar{x})) < \theta \tag{12c}$$

$$lim_{t\to+\infty} e^{-rt}\mu(t) \geq 0, \; lim_{t\to+\infty} e^{-rt} q(t)\mu(t) = 0. \tag{13}$$

Recall that $v = H(f) - y$ is the virgin input for producing paper, and $w = \delta(q + \bar{x})$ is the recycled input.

The costate variable μ is the implicit social price of one unit of paper. It represents the additional social benefit to society from an increase of one unit in paper consumption.[7] Thus, the variable μ is nonnegative ($\mu \geq 0$). Equation (10) is an intertemporal efficiency condition for the shadow price of paper. Its right-hand side represents the marginal social benefit of paper, the additional benefit to society from a one-unit increase in paper consumption. This marginal social benefit equals the benefits that consumers gain by consuming one additional unit of paper (the marginal utility of consumers), $u'(c)$, minus the costs incurred. The latter is the sum of the price paid by consumers to obtain the good (μ), the marginal net cost of recycling the fraction δ ($C_w(w) - \mu G_w(v, w)$), and the marginal cost of the fraction of paper that is disposed of in the landfill as waste ($(1-\delta)\theta$). The net recycling cost equals the curbside recycling cost $C_w(w)$ minus the additional production gain when the recycling part is used as an input in the production process ($\mu G_w(v, w)$). The left-hand side of Equation (10) measures the opportunity cost of maintaining production at any point in time. It

[7] $\mu^*(t) = \partial W(q^*(t), f^*(t), \delta^*(t))/\partial q * (t)$ where $(q^*(t), f^*(t), \delta^*(t))$ is the optimal solution.

includes both the interest charge ($r\mu$) and the unit gains ($-\dot{\mu}$). Therefore, Equation (10) states that society should consume paper to the point where the marginal social benefit of paper is equal to the social cost of this capital.

Equation (11) defines the optimal land allocation. It states that society should allocate land to equalize the marginal net benefits of both competing land uses. The left-hand side of this equation represents the marginal benefit of forest products. It is the sum of the marginal benefit of forest activities (timber) and its marginal external benefit. The right-hand side of Equation (11) represents the marginal net benefit of the other land use (agriculture).

Equations (12a)-(12c) define the optimal recycling rate. As we explained above, the term $C_w(w) - \mu G_w(v, w)$ is the net marginal cost of recycling. If the marginal cost of the landfill is greater than the cost of recycling, optimality requires throwing all waste paper into the landfill, since recycling is more costly than landfilling. However, if the net marginal cost of recycling is less than the marginal cost of landfilling, optimality requires complete recycling ($\delta = 1$). Finally, equation (12b) indicates that, for an interior solution, the recycling rate must be chosen so that the net marginal cost of recycling equals the marginal cost of landfilling.

Finally, Equation (13) is the transversality condition. To characterize the optimal solution of the problem of (4)-(8), let us first focus on the interior solution for the optimal recycling rate. Then, substituting the net marginal recycling cost $C_w(w) - \mu G_w(v, w)$ from Equation (12b) into Equation (10) gives

$$r\mu - \dot{\mu} = u'(c) - \theta - \mu. \tag{14}$$

As we show in the Appendix, Equation (11) implies that both f and $w = \delta(q + \bar{x})$ are functions of μ.

Therefore, the system's motion in the phase space (q, μ) is governed by the following two differential equations, with one initial and one terminal condition. They define a potentially optimal path $[q^*, \mu^*]$ characterized by [8]

$$\dot{q} = G(\psi_1(\mu) - y, \psi_2(\mu)) - q \tag{15}$$

$$\dot{\mu} = -u'(q + \bar{x}) + \theta + (1 + r)\mu \tag{16}$$

where $H(f) = \psi_1(\mu)$ and $\delta(q + \bar{x}) = \psi_2(\mu)$ are increasing functions of μ. Both functions are defined in the Appendix as well as their derivatives with respect to μ. If we assume that the exogenous variables y and θ converge to the steady states $\bar{y}$ and $\bar{\theta}$, respectively, then the steady state of (q, μ) from equations (15)-(16) defined by $\dot{q} = \dot{\mu} = 0$ exists and is unique (see Appendix). This is illustrated in Figure 1, which also shows the optimal solution in the phase space (μ, q). The phase arrows in regions I-IV in Figure 1 indicate how q and μ move in these regions, i.e., the signs of $(\dot{q}, \dot{\mu})$. Figure 1 shows that the steady state is unstable. According to the Figure, given $q(t_0) = q_0$, $\mu(t_0)$ must be chosen in regions III or I, and according to a well-known theorem on differential equations (see, e.g., Pontryagin (1962)), there exists exactly one trajectory that converges to $\dot{q} = \dot{\mu} = 0$. This solution satisfies the transversality condition and is optimal. This is shown in Figure 1 for both starting points $q(t_0)$ at low and high values.

Analyzing some static comparisons in the steady state would be interesting. The analysis is helpful to the regulator in implementing incentive policies. As shown in the Appendix $\dot{q} = 0$ and $\dot{\mu} = 0$, it is equivalent to $\mu = L_1(q)$ and $\mu = L_2(q)$, where the functions L_1 and L_2 depend on q and other swift parameters. The function $L_2(q) = [u'(q + \bar{x}) - \bar{\theta})]/(1 + r)$ implies that $\partial L_2/\partial r < 0$. L_1 does not depend on r. The curve L_2 shifts down so that both μ^* and q^* decrease. Given that f and w are increasing functions of μ, then both f^* and w^* are decreasing. The effect on $\delta^* = w^*/(q^* + \bar{x})$ is undetermined. We can then conclude that:

$$(a)\ \frac{\partial f^*}{\partial r} < 0 \quad (b)\ \frac{\partial q^*}{\partial r} < 0 \quad (c)\ \frac{\partial w^*}{\partial r} < 0 \tag{17}$$

This result indicates that the higher the interest rate, the lower the land area allocated to forestry, the lower the paper production, and the lower the quantity of recycled input.

[8] Both q^* and μ^* depend on time t.

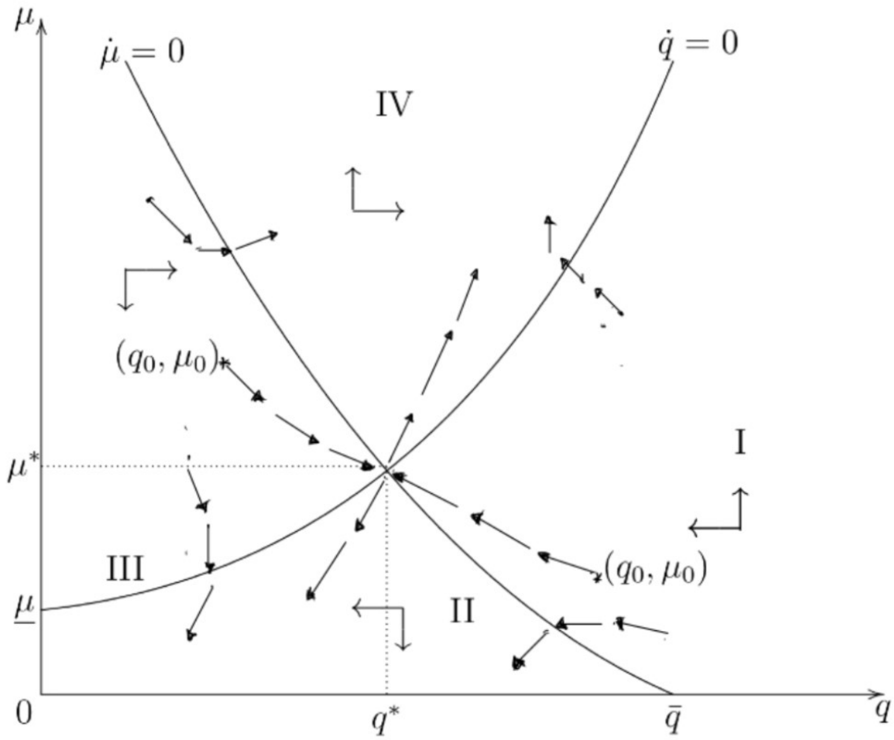


Figure 1: The optimal solution

However, the effect on the social recycling rate is undetermined.

$$(a)\ \frac{\partial f^*}{\partial \bar{\theta}} < 0 \quad (b)\ \frac{\partial q^*}{\partial \bar{\theta}} < 0 \quad (c)\ \frac{\partial w^*}{\partial \bar{\theta}} > 0 \quad (d)\ \frac{\partial \delta^*}{\partial \bar{\theta}} > 0 \tag{18}$$

The intuition behind the results (18) is as follows. An increase in landfill costs implies a higher marginal social cost of consuming an additional unit of paper. Therefore, optimality requires reducing the quantity of paper. On the other hand, the net marginal recycling cost is relatively lower than the landfill cost, so recycling is preferable. It is optimal to first substitute virgin inputs with recycled inputs, thereby reducing the land area devoted to forests.

2.2 Market Equilibrium

We derive the competitive equilibrium quantities for the virgin wood market used by the paper manufacturing industry, as well as for the markets for paper and agricultural goods used for household consumption.

The manufacturing industry of paper: The representative paper production firm does not consider the social costs of recycling and non-recycling. Given the recycling rate δ, the price of final paper production p, the price of virgin input p^v, and the price of recycled input p^w (all of which are time-dependent), the representative firm's problem involves choosing the demand for inputs v and the supply of outputs q to maximize discounted profits. The problem is then written as:

$$\Pi = \max_{q,\, v} \int_{t_0}^{+\infty} [pq - p^v v - p^w \delta(q + \bar{x})] e^{-rt} dt \tag{19}$$

subject to

$$\dot{q} = G(v, \delta(q + \bar{x})) - q \tag{20}$$

$$q \geq 0, \; q(t_0) = q_0. \tag{21}$$

The current value Hamiltonian is $\mathcal{H}(q, v) = pq - p^v v - p^w \delta(q + \bar{x}) + \lambda(G(v, \delta(q + \bar{x})) - q)$ where λ is the costate variable that depends on the time t. The first order condition for optimality requires that

$$r\lambda - \dot{\lambda} = p - \delta p^w + \lambda(\delta G_w(v, \delta(q + \bar{x})) - 1) \tag{22}$$

$$p^v = \lambda G_v(v, \delta(q + \bar{x})) \tag{23}$$

The demand for virgin input v and the supply of paper q are, therefore, solutions of the system given by (22)-(23).

The logging industry: We assume that the logging industry does not account for the positive externalities generated by forests. The problem of the representative logging firm is static. At each time t, given the price of virgin input p^v (which, at market equilibrium, is the same as the price of virgin wood), the firm chooses the planting area f that determines the total area of forest sf and the land area devoted to agriculture a to maximize its profit, $p^v H(f) + B^a(a) - kf$ subject to (2), where k represents the planting costs for forestry and agriculture. Substituting $a = A - sf$ from Equation (2) into the profit function, the first order necessary condition with respect to f gives $p^v H_f(f) - sB^a_a(A - sf) - k = 0$, such that

$p^v = \dfrac{k + sB_a^a(A - sf)}{H_f(f)}$. The equilibrium price implies that we can substitute this expression of p^v into Equation (23) above to obtain the results we need.

$$\lambda = \frac{k + sB_a^a(A - sf)}{G_v(v, w)H_f(f)}. \tag{24}$$

Equations (22) and (24) determine the market equilibrium for virgin wood.

The representative consumer:[9] At each time t, the consumer chooses both the consumption of paper $c = q + \bar{x}$ (which in equilibrium is the same as the quantity of paper produced by the manufacturing industry net of imports) and agricultural products z (which in equilibrium is the same as the agricultural products produced by the logging industry $B^a(a)$), which maximizes its utility $u(c) + z$ under the budget constraint $pc + p^a z = I$, where p^a is the market price of the good z, and I is the income of the consumer. Here, we have assumed that the price of imported paper is the same as that produced domestically. Substituting z from the budget constraint into the objective function and differentiating it with respect to q gives:

$$u'(c) = \frac{p}{p^a} \tag{25}$$

Equations (22), (24), and (25) determine the market equilibrium for paper and agricultural goods, given the market prices p and p^a. All of these results define the equilibrium outcome of the market as the allocation (f, v, q, z) that satisfies the system of equations (22), (23), (24), and (25), given the recycling rate δ, the market prices (p, p^v, p^w, p^a), and the discount rate r.

The competitive equilibrium would not be Pareto optimal, at least because the market needs to account for the positive externalities of forests and the social costs of recycling. Even if the social recycling costs are imposed on the paper manufacturing industry, the price for the positive externalities of forests would still be pending. Therefore, the implied recycling rates stemming from the competitive equilibrium and the social optimum would differ. Whether one is higher than the other is an empirical question that we examine in the following sections

[9] All variables are time-dependent except for $\bar{x}$, which remains constant.

using data from the U.S.

3 Empirical Analysis

The U.S. offers an ideal empirical framework for this study, given that 56% of its forest land is privately owned. Furthermore, between 1965 and 2015, which is our data range, 78.43% of the planted forest area was planted by private landowners.[10] As the U.S. Department of Agriculture (USDA) pointed out, the planted forest area is a response to the demand for forest products and competing non-timber uses.

3.1 Econometric specification

Our objective is to estimate the model based on the market equilibrium conditions. Recall that these conditions can be summarized from the system of equations (22)-(23) obtained above, given by:

$$\begin{aligned} r\lambda - \dot{\lambda} &= p - \delta p^w + \lambda[\delta G_w(v,w) - 1] \\ \lambda &= \frac{k + sB_a^a(A - sf)}{G_v(v,w)H_f(f)} \end{aligned}$$

For analytic tractability, we make assumptions about the parametric functional forms of the paper production function $G(v,w)$, the virgin wood production function $H(f)$, and the benefit function of other competing uses of land $B^a(a)$. This can only be done under prevailing market conditions, without government intervention. Let $(q(t), f(t), h(t), \delta(t), v(t))$ be the observable outcomes generated at period t.

Bongers and Casas (2022) use a general CES function with constant returns to scale to model the production function at the macroeconomic level, using virgin natural resources and recycled materials. We follow these authors by using a CES function to model paper production as a function of virgin input v and recycled paper input w. This function

[10] Source: U.S. Forest Service and Private Forestry Tree Planters' Notes.

is specified as $G(v,w) = (\beta_1^g v^\sigma + \beta_2^g w^\sigma)^{1/\sigma}$, where $(\sigma-1)/\sigma$ is the elasticity of substitution, measuring the degree to which one input can be substituted for another. Because we suspected high substitutability between virgin wood and recycled paper, We tested the hypothesis $\sigma = 1$, which was supported by the data, suggesting a particular case of the CES function of the form $G(v,w) = \beta_1^g v + \beta_2^g w$. This means that if the paper industry wants to increase, for instance, a unit of recycled paper while maintaining its production constant, it will decrease the virgin input at a constant rate of β_2^g/β_1^g. Preliminary results show that this production function provides a nearly perfect fit with the data by explaining more than 99% of the variability of the final paper output (see later in Table 2 and Figure A.1 in the Appendix). In addition, this production function provides analytical tractability for solving the differential equation involved in the problem. In the remainder of the analysis, we will present the empirical testing procedure with this production function.[11] To model the forest industry's production function, we account for natural regeneration and other factors that can change forest production without altering the planting area at time t. We define $H(f) = \beta_1^f f + \beta_0^f$, where $H(0) = \beta_0^f$ represents production without planting. We also assume that $B^a(a) = \beta_0^a a - \dfrac{\beta_1^a}{2} a^2$ and a quadratic utility function $u(c) = \beta_0^c c - \dfrac{\beta_1^c}{2} c^2$.

Substituting these functions into (22) and (24), we obtain the following equilibrium conditions

$$[\delta\beta_2^g - 1 - r]\beta_0 + s^2[(\delta\beta_2^g - 1 - r)f + \dot{f}]\beta_1^a = -[p - \delta p^w]\beta_1^g\beta_1^f \qquad (26)$$

where $\beta_0 = k + s(\beta_0^a - \beta_1^a A)$.

To estimate the model, the equations and equilibrium conditions obtained can only be assumed to hold in expectation, given the uncertainty in the real data. The econometric model

[11]We also explored the CES production with decreasing returns to scale specified as $G(v,w) = (\beta_1^g v^\sigma + \beta_2^g w^\sigma)^{\rho/\sigma}$, where ρ is the degree of homogeneity of the function or the return to scale, $\rho \le \sigma \le 1$. Our data provided an excellent fit and supported the hypotheses $\rho = \sigma^g = 1$ against the alternatives $\rho < \sigma^g < 1$, suggesting that, in a competitive industry such as logging, it is challenging for firms to operate with decreasing returns to scale technologies.

can therefore be specified as a system of moment restrictions defined by:

$$\mathbb{E}_t[\boldsymbol{m}(\boldsymbol{z}_t, \boldsymbol{\beta})] = 0 \tag{27}$$

where $\boldsymbol{z}_t = [f_t, \Delta f_t, h_t, q_t, w_t, v_t, p_t, p_t^w, p_t^a, c_t, \delta_t, r_t]$ is the vector of variables (available in the data), $\boldsymbol{\beta} = (\beta_0^f, \beta_1^f, \beta_1^g, \beta_2^g, \beta_0^a, \beta_1^a, \beta_0^c, \beta_1^c)$ is the vector of model parameters, and the $L \times 1$ vector of moment functions is given by $\boldsymbol{m}(\boldsymbol{z}_t, \boldsymbol{\beta}) = [m_1(\boldsymbol{z}_t, \boldsymbol{\beta}), m_2(\boldsymbol{z}_t, \boldsymbol{\beta}), \ldots, m_L(\boldsymbol{z}_t, \boldsymbol{\beta})]$. The values of the constants k, s, and A are taken from the data as discussed later, and $\mathbb{E}_t[\cdot]$ is the expectation given the information available at time t. As explained in the Appendix, the L moment functions that are used for estimation are built from the following 4 structural moments obtained from the above theoretical results:

$$m_1(\boldsymbol{z}_t, \boldsymbol{\beta}) = \beta_1^f f_t + \beta_0^f - h_t \tag{28}$$

$$m_2(\boldsymbol{z}_t, \boldsymbol{\beta}) = \beta_2^g + \beta_1^g \frac{v_t}{w_t} - \frac{q_t}{w_t} \tag{29}$$

$$m_3(\boldsymbol{z}_t, \boldsymbol{\beta}) = [\delta_t \beta_2^g - 1 - r_t]\beta_0 + s^2[(\delta_t \beta_2^g - 1 - r_t) f_t + \triangle f_t]\beta_1^a + [p_t - \delta_t p_t^w]\beta_1^g \beta_1^f \tag{30}$$

$$m_4(\boldsymbol{z}_t, \boldsymbol{\beta}) = \beta_0^c - \beta_1^c c_t - \frac{p_t}{p_t^a} \tag{31}$$

The coefficients on the variables are connected through cross-coefficient restrictions imposed by the system of structural estimating equations derived from the theory and the specific forms of the utility and production functions we have chosen. For the estimation, a backward Euler discretization approach is used such that the continuous derivative of f_t is replaced by its difference $\Delta f_t = f_t - f_{t-1}$, and we assume that the expectational structure that enforces these restrictions is "soft" so that cross-equation restrictions need not be explicit. The details of the Generalized Method of Moments (GMM) estimation that we use are provided in the Appendix.

3.2 Data

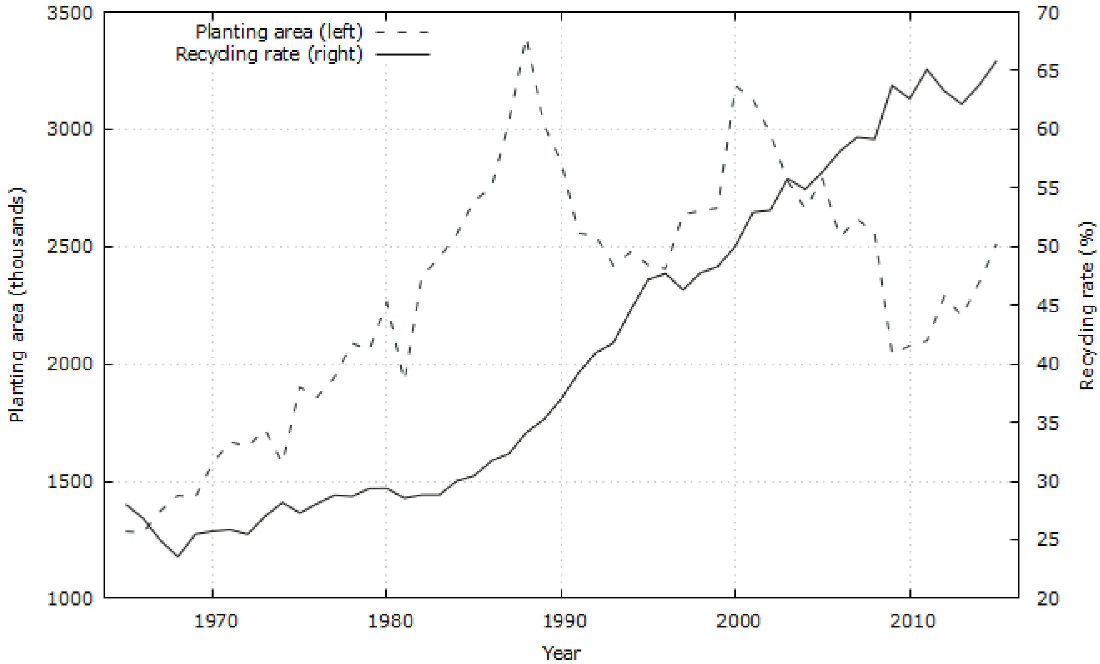
To estimate the parameters of production functions for wood, paper, and agriculture (other competing uses of land), we use data provided by the USDA Forest Service, which can also be found in Howard and Lian (2019), except for the average annual planting area of timber, which can be found in Oswalt et al. (2019). The data range from 1965 to 2015, and the dollar amounts are expressed in 2009 dollars. (2019). The data range from 1965 to 2015, and the dollar amounts are expressed in 2009 dollars. The data range from 1965 to 2015, and the dollar amounts are expressed in 2009 dollars. The data range from 1965 to 2015 and pertain to the United States. Table 1 presents the descriptions and units of the variables, as well as their summary statistics.

Table 1: Variable Description and Summary Statistics (Annual Data)

Var	Description	Unit	Mean	Std.Dev.	Min	Max
f_t	Area of timber land planted	Million acres	2.3089	0.5275	1.2808	3.3938
h_t	Production of forest product	Million tons	155.12	27.936	108.26	197.42
q_t	Production of paper	Million tons	72.737	16.791	40.489	97.020
c_t	Consumption of paper	Million tons	78.959	16.435	48.270	105.32
v_t	Pulpwood production	Million tons	54.567	8.3290	33.993	67.103
w_t	Recovered paper	Million tons	31.796	16.010	10.904	54.323
δ_t	Recycling rate	Proportion	0.4100	0.1400	0.2350	0.6600
p_t	Price of paper	Price index	94.780	21.020	46.200	120.00
p_t^w	Price of recovered paper	Price index	72.280	29.730	21.780	121.14
p_t^a	Price of agricultural product	Price index	58.480	29.850	17.500	133.17
r_t	Interest rate	Proportion	0.0400	0.0230	0.0130	0.0860

The recycling rate ranges from 20% to 66%, with an average rate of 36%, while the planting area ranges from 1.28 to 3.39 million acres, with an average of 2.3 million acres. Figure 2 shows the evolution of the planting area (vertical left axis) and the recycling rate (vertical right axis) over time. The data show that the recycling rate has been steadily increasing over time, with a more drastic pace in recent years (1985-2015). In contrast, the forest planting area has fluctuated somewhat, with a downward trend since the mid-1980s,

Figure 2: **Evolution of planting area and recycling rate over time**

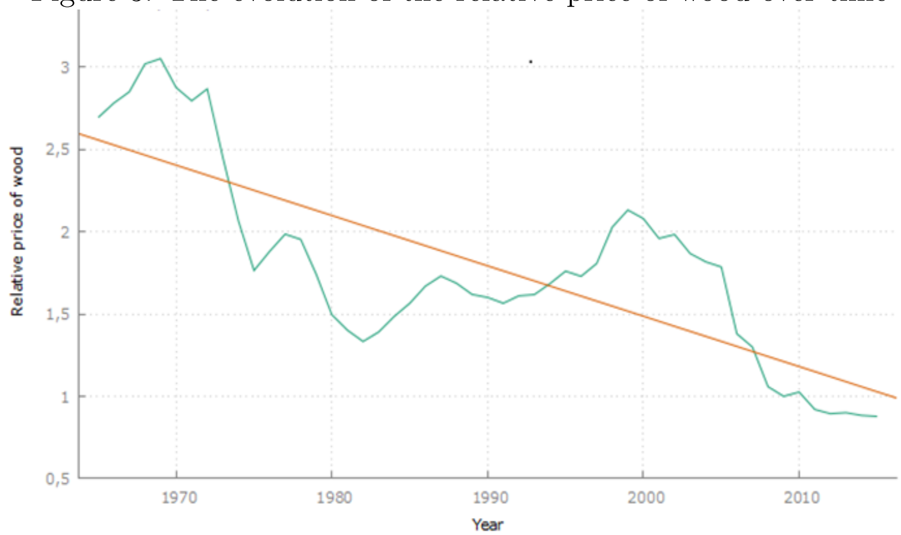


peaking around 1987 and 2000. This might suggest that optimal recycling, which accounts for the net social benefit, has been reached.

It is also important to assess whether the value of wood has decreased as recycling has grown over the years. To answer this question, we examine the trend in the relative price of wood, as forest companies consider the profits from both forest activities and alternative land uses in their decisions. As shown in Figure 3, the relative price of wood, calculated as the price of wood divided by the price of alternative land uses (i.e., agriculture), has generally decreased over time, with an average decline of 22.8% between 1965 and 2000, and an additional average decline of 57.8% between 2000 and 2015. This trend suggests that alternative land uses, such as agriculture, have become more profitable than timber production. As a result, private forest companies are more inclined to allocate land to agriculture than to replant trees, consistent with the excessive recycling hypothesis developed in this study.

The optimal recycling rate is computed empirically by estimating the parameters necessary to quantify the solutions of our theoretical model. We do this using the functional

Figure 3: The evolution of the relative price of wood over time



forms for both utility and production as well as the estimation approach discussed above.

3.3 Estimation of parameters

We estimated the production and demand functions of the firms summarized by the vector of model parameters $\boldsymbol{\beta}$ using the equilibrium market data described above. We then applied GMM estimation to the vector of moment restrictions (27) given by the system of Equations (28)-(31), as described in the Appendix. Table 2 summarized the GMM estimation results. All the estimated parameters have the expected signs and are statistically significant at the 5% level. The J-statistic is estimated at 4.52, which is less than the critical value of 9.488, indicating a very good fit of the model with the data at a 5% significance level.

Now, let us turn to the estimation of external benefits and costs that the market does not take into account. In the forest management problem, the calculation of carbon sequestration benefits is endogenously determined by tree age; see, e.g., Kooten et al. (1995) and Mendelsohn and Sohngen (2019). However, as highlighted in the previous section, given our limited dataset, this framework cannot be used. Instead, we use U.S. annual data from 1990 to 2015 from FAOSTAT, which includes (i) the Adjusted savings of carbon dioxide damage (current US$), (ii) the CO2 emissions (in gigagrams (Gg)), and (iii) the total CO2 stock

Table 2: **GMM estimates**

Parameters	Name	Estimate	Std. Err.
Forest production function	β_1^f	0.0450	0.0040
	β_0^f	52.321	9.6959
Paper production function	β_1^g	0.9650	0.0100
	β_2^g	0.6310	0.0240
Marginal benefit of agriculture	β_1^a	0.8349	0.0126
	β_0^a	0.7370	0.1920
Consumer marginal utility	β_0^c	3.5700	0.3610
	β_1^c	0.0210	0.0045
J-statistic		4.5213	

in the forest. We first compute the average carbon dioxide damage (current US$/Gg) by dividing the adjusted savings by the CO2 emissions. Then we multiply this by the total CO2 stock in the forest to obtain the total carbon benefit. Finally, we convert this value to 2009 dollars using the U.S. consumer price index. To estimate the external benefit of the forest as a function of the average annual planting area, we assume that $B^e = \beta_0^e f - \beta_1^e f^2$, where B_t^e is available in the data over the study period. Table 3 presents the result of the regression estimation. All coefficients are significant, and the high R^2 of 82.7% indicates that the functional form provides a very good fit to the data.

To estimate the recycling cost, we use Kinnaman's (2010) translog cost function to construct a pseudo cost.[12] That is, we assume a U-shaped quadratic cost function such that the marginal cost can be written as $C_w(q) = \gamma_1 q + \gamma_0$. A linear marginal cost function provides a good approximation and simplifies the resolution of the problem governed by a system of differential equations. Table 3 presents the estimates. For the marginal landfill

[12]The translog cost function is given by $ln(c) = d_0 + d_1 ln(q) + d_2(ln(q))^2$. Kinnaman (2010) estimated $d_0 = 7.792$, $d_1 = 0.321$, and $d_2 = 0.033$. The pseudo marginal cost is then $C_w(q) = c(d1 + 2d_2 ln(q))/q$. This is used to estimate the linear marginal cost $\gamma_1 q + \gamma_0$.

Table 3: **Estimates of External Costs and Benefits**

Parameters	Name	Estimate	Std. Err.
Marginal external benefit of forest	β_0^e	0.014	0.001
	β_1^e	2.9930	0.2792
Marginal recycling cost	γ_0	70.093	0.0510
	γ_1	0.1371	0.0007
	R^2	0.827	

cost, we assume that $\theta(t) = \bar{\theta} + (\theta_0 - \bar{\theta})e^{-gt}$, where g is the rate at which the cost converges to the steady state $\bar{\theta}$. This implies that total landfill costs increase cumulatively over time; that is, more than proportionally with increased waste disposal as landfill space becomes scarcer and environmental regulations stricter. We used the median value of $\bar{\theta} = 59.70\$/ton$ that Kinnaman estimated from municipal collection in the US. We determined the value of $g = 0.0767 = 7.67\%$ using U.S. data from the National Solid Wastes Management Association from 1980 to 2013.

3.4 Results

To solve the estimation problem, we assume that $A = 900$ million acres. The steady-state quantity of virgin wood produced in other sectors is $\bar{y} = 91.3$ million tons, the net import $\bar{x} = 4$ million tons representing the median. Based on the previously estimated value, $s = 119.64$, and $k = 2.054$, the problem (4)-(8) is solved analytically.[13] The socially optimal steady-state of the recycling rate (δ), the planting forest area (f), the stock of paper (q), and the recycling input (w) are shown in Table 4 for different values of the social discount rate (r). The discount rate ranges from $r = 3\%$ to $r = 10\%$. The steady-state optimal recycling rate is approximately 58%. As seen in Table 4, the steady-state social outcome is

[13]The value of the parameter s is the mean of the ratio of the total forest area, including the natural regeneration of planted areas over time, to the area of forest planted, using data from FAOSTAT and Oswalt et al. (2019) from 1990 to 2015.

less sensitive to the interest rate.

Table 4: Steady-state results

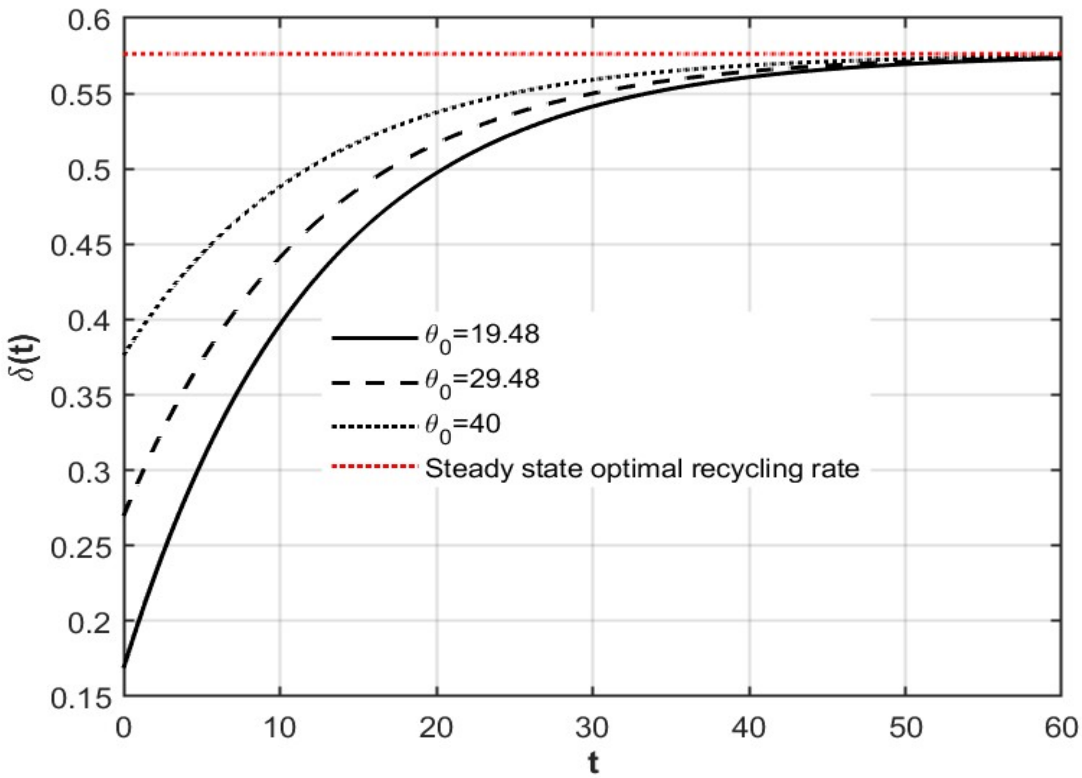
r(%)	δ(%)	f(million)	q (million)	w (million)
3	57.7%	2.2311	0.0928	0.0547
5	57.6%	2.2268	0.0924	0.0544
8	57.5%	2.2205	0.0918	0.0539
10	57.4%	2.2162	0.0914	0.0535

Figure 4 illustrates the optimal trajectory of the recycling rate in the U.S. and its evolution over time, measured in years. The figure presents the optimal path for various initial marginal disposal cost values, denoted as $\theta_0 > 0$.[14] As observed, the optimal path exhibits a consistent increase and monotonically converges to the steady state, which is 57.6% to reach within 60 years.

Figure 5 displays both the observed recycling rate patterns and the optimal recycling rate from 1980 to 2015. The starting year of 1980 is selected to ensure the correct trajectory, as the marginal disposal cost value begins in 1980 with a specific value of $\theta_0 = 19.48$. This value is used as the initial marginal disposal cost. It is important to note that any realized recycling rate below the curve of the optimal path indicates insufficient recycling, while a rate above the optimal path suggests excessive recycling. In particular, we note that (i) the observed recycling rate is closer to the optimal rate from the early 1980s to the late 2000s; (ii) high recycling rates in recent years exceed the optimal rate, particularly after 2000; and (iii) the last 15-20 years of projections have experienced increasing gaps between the actual rate and the optimal rate. Consequently, as depicted in the figure, the economy has been engaging in excessive recycling over the past two decades. More precisely, the realized rate has exceeded the optimal path since 2000 and has surpassed the optimal long-term value of

[14]The graphic is depicted with an initial value of $q_0 = 40489$, representing the fifth centile. However, for a stable branch, as solved in the model, the initial value does not impact the optimal path.

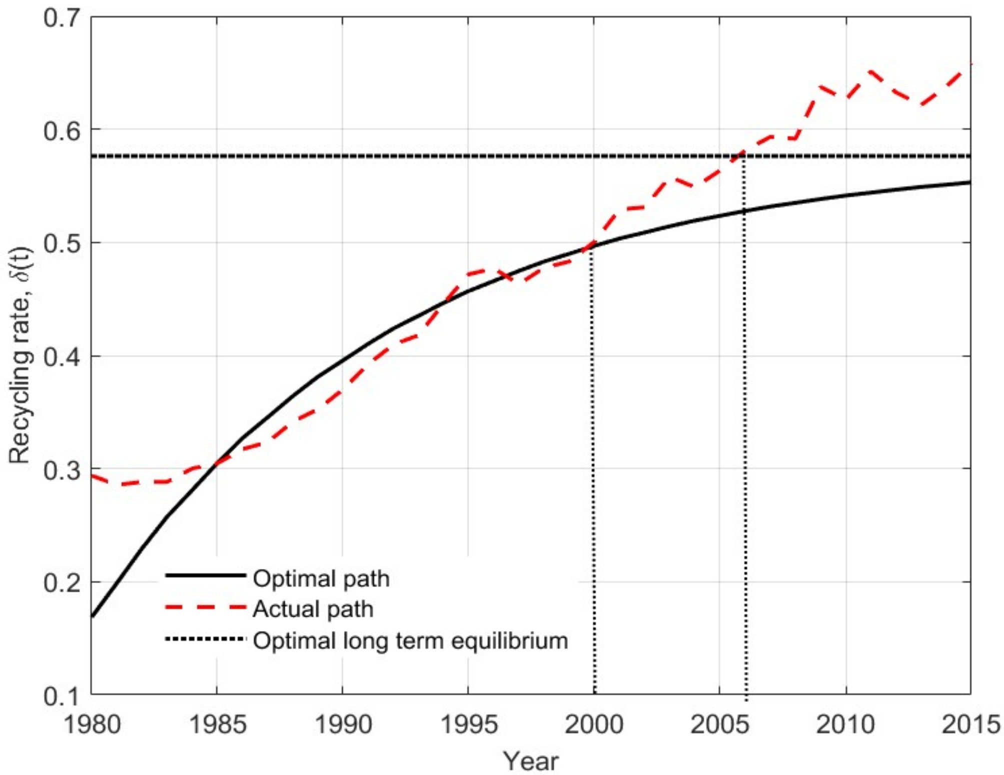
Figure 4: Optimal recycling path (r=5%)



57.6% since 2006.

As noted in the theoretical model, the optimal recycling rate depends on the net recycling cost and the landfill cost, where the net recycling cost is the curbside recycling cost minus the additional production gain from using the recycled material as an input in the production process. Therefore, it would be interesting to analyze the sensitivity of the socially optimal outcome relative to changes in landfill or recycling costs. We perform a steady-state sensitivity analysis. Table 5 shows how the steady state social outcome changes relative to a change of up to 15% of the marginal external benefit of forests (MBF), the marginal landfill cost (θ), and the marginal recycling cost (parameter γ_0). It follows from Table 5 that the optimal steady state recycling rate and the forest area are less sensitive to changes in the marginal benefit of the forest than to changes in the marginal cost of landfill or the marginal cost of recycling. For example, a 10% increase in the marginal external benefit of the forest leads

Figure 5: Realized recycling rate and optimal recovery rate ($\theta_0 = \$19.48$)



to a decrease of 0.23% in the recycling rate and an increase of 0.1% in the forest area. The recycling rate and the forest change by -35.6% and 13.5%, respectively, when the marginal landfill cost decreases by the same magnitude (10%). This suggests that a fall in social cost has a much greater impact on society, in terms of social benefits and the preservation of forest areas, than any incentive policy that induces an increase in amenity benefits of the same magnitude.

Table 5: Steady-state: sensitivity analysis

	Change in MBF		**Change in θ**		**Change in γ_0**	
$\triangle$(%)	$\triangle\delta$	$\triangle f$	$\triangle\delta$	$\triangle f$	$\triangle\delta$	$\triangle f$
-15%	0.35%	-0.05%	-52.8%	20.1%	61.5%	-20.1%
-10%	0.23%	-0.10%	-35.3%	13.5%	41.9%	-13.5%
-5%	0.12%	-0.15%	-17.7%	6.7%	21.4%	-6.7%
5%	-0.12%	0.05%	17.8%	-6.7%	-22.5%	6.7%
10%	-0.23%	0.10%	35.6%	-13.4%	-46.1%	13.5%
15%	-0.35%	0.15%	53.5%	-20.2%	-71.0%	20.2%

4 Conclusion

This paper presents a dynamic equilibrium model between the paper manufacturing sector and the logging industry to investigate the long-run impact of recycling on forest inventory. By incorporating the non-market externalities of forest standing stocks alongside the social costs of waste management, we characterize a socially optimal recycling trajectory. Our central finding reveals a critical threshold in the recovery of secondary fibers: beyond a certain point, increased recycling suppresses virgin wood prices to a level that triggers land-use conversion away from forestry. Empirical calibration using USDA Forest Service data (1965-2015) suggests that the socially optimal recovery rate converges to approximately 58%. Because observed U.S. recovery rates have consistently exceeded this threshold, our results indicate that current policy intensity may be inadvertently reducing the very forest stocks it seeks to preserve by eroding the economic viability of private timberland investment.

The mechanism driving this paradox is the shift in the opportunity cost of land. While nominal timber prices have fluctuated, the expansion of secondary fiber markets has decelerated price growth for virgin wood relative to competing land uses, such as agriculture. This decline in the relative return on forestry incentivizes land-use conversion, ultimately reducing the total acreage devoted to timber stands. From a policy perspective, our findings do not suggest that recycling lacks merit; rather, they emphasize the necessity of marginal analysis. Recycling remains socially optimal as long as the marginal cost of recovery is offset by the combined value of avoided disposal externalities and the marginal social benefit of forest preservation. However, our results demonstrate that once the recycling rate surpasses the 58% threshold, the marginal social cost of lost forest inventory exceeds the marginal gains from waste diversion, resulting in a net welfare loss.

These results suggest that environmental policy should prioritize the maintenance of optimal forest land-use over the pursuit of unrestricted recycling mandates. Rather than pushing for universal recovery targets, policymakers should calibrate incentives to reflect the marginal social benefits of forest inventory. Furthermore, our model indicates that public investment in R&D to reduce the marginal social costs of sorting and processing, thereby shifting the optimal recovery threshold outward, is a more efficient mechanism for forest conservation than direct subsidies for amenity benefits. Ultimately, achieving a truly efficient circular economy requires a shift away from volume-based recycling goals toward a dynamic framework that targets optimal recovery rates while accounting for the endogenous responses of primary resource markets

Appendix

Existence and uniqueness of the steady state

The equation (11) that defines the optimal allocation of the forest depends on f, μ, δ, and q. Substituting $\delta(q+\bar{x}) = w$, the implicit function theorem implies that $f = \psi(\mu, w)$ is a continuous and differentiable function of μ and w. In addition, totally differentiating Equation (11) with respect to f, μ, and w, we get

$$\psi_\mu(\mu, w) = -\frac{H_f(f)G_v(v,w)}{s^2 B^a_{aa}(a) + B^e_{ff}(f) + \mu H_{ff}(f)G_v(v,w) + \mu H_f^2(f)G_{vv}(v,w)} > 0 \quad \text{(A.1)}$$

$$\psi_w(\mu, w) = -\frac{\mu H_f(f)G_{vw}(v,w)}{s^2 B^a_{aa}(a) + B^e_{ff}(f) + \mu H_{ff}(f)G_v(v,w) + \mu H_f^2(f)G_{vv}(v,w)} > 0. \quad \text{(A.2)}$$

Substituting $f = \psi(\mu, w)$ into Equation (12b), which defines the optimal recycling rate for an interior solution, we get $-C_w(w) + \theta + \mu G_w(H(\psi(\mu, w)) - \bar{y}, w) = 0$ By the implicit function theorem, we conclude that $w = \psi_2(\mu)$ is a function of μ. Differentiating this equation with respect to w and μ gives:

$$\psi_{2\mu}(\mu) = -\frac{G_w(v,w) + H_f(f)G_{vw}(v,w)\psi_\mu}{-C_{ww}(w) + \mu(G_{ww}(v,w) + H_f(f)G_{vw}(v,w)\psi_w(\mu,w))} > 0. \quad \text{(A.3)}$$

Finally, substituting $\psi(\mu, w)$ and $\psi_2(w)$ into Equation (5), we have $\dot{q} = G(H(\psi(\mu, \psi_2(\mu))) - \bar{y}, \psi_2(\mu)) - q$. Let $\psi_1(\mu) = H(\psi(\mu, \psi_2(\mu)))$ with $\psi_{1\mu} = (\psi_\mu(\mu, w) + \psi_{2\mu}(\mu)\psi_w(\mu, w))H_f(\psi(\mu, \psi_2(\mu))) > 0$; the necessary conditions for an optimum are

$$\dot{q} = G(\psi_1(\mu) - \bar{y}, \psi_2(\mu)) - q = \Gamma_1(q, \mu) \quad \text{(A.4)}$$

$$\dot{\mu} = -u'(q+\bar{x}) + \theta + (1+r)\mu = \Gamma_2(q, \mu). \quad \text{(A.5)}$$

At the steady state $\dot{q} = \dot{\mu} = 0$. Therefore, the implicit function theorem implies that $\mu = L_1(q)$ and $\mu = L_2(q)$ in Equations (A.4) and (A.5), respectively. The steady state is the intersection of functions $L_1(q)$ and $L_2(q)$. Let us prove that this steady state exists and is unique. First, we have that: $L_{1q}(q) = 1/[\psi_{1\mu}(\mu)G_v(v,w) + \psi_{2\mu}(\mu)G_w(v,w)] > 0$ and $L_2(q) = u''(c)/(1+r) < 0$. Thus, function $L_1(q)$ is strictly increasing in q, while function $L_2(q)$ is strictly decreasing in q. Second, the function $L_2(q)$ intersects the q-axis at $\bar{q} = (u')^{-1}(\theta) - \bar{x} > 0$. Turning to function $L_1(q)$, since $\Gamma_1(q,\mu) = G(v,w) - q$ where $w = \delta q = \psi_2(\mu)$ and $v = \psi(\mu)$, $\Gamma_1(0,\mu) = G(v,0) \geq 0$. Since $G(0,0) = 0$, then $v = H(f) - \bar{y} = 0$, implying that $f = H^{-1}(\bar{y})$ and $\mu = \psi_1^{-1}(H^{-1}(\bar{y})) \geq 0$. The previous statement shows that the functions $L_1(q)$ and $L_2(q)$ intersect at a single point, thereby establishing the existence and uniqueness of the steady state.

Sufficient condition

We demonstrate the sufficient condition for the optimal trajectory path in the social planner problem (4)-(8). To simplify, we express the Hamiltonian (9) as a function of (q, f, w).

$$\mathcal{H}(q,f,w) = U(q+\bar{x}, B^a(A-sf)) + B^e(f) - C(w) - \theta(q-w) + \mu(G(H(f)-y,w)-q).$$

We then prove the joint concavity of the Hamiltonian $\mathcal{H}(q,f,w)$ at (q,f,w). $\mathcal{H}(q,f,w)$ is concave if the Hessian matrix of second-order partial derivatives is negative semi-definite. The second-order partial derivatives are:

$\mathcal{H}_{qq} = U_{qq}(c) = u''(c) < 0; \mathcal{H}_{qf} = \mathcal{H}_{qw} = 0;$

$\mathcal{H}_{ff} = s^2 B^a_{aa}(A-sf) + \mu H_{ff}(f)G_v(v,w) + \mu(H_f(f))^2 G_{vv}(v,w) < 0;$

$\mathcal{H}_{fw} = \mu H_f(f) G_{vw}(v,w) > 0;\ \mathcal{H}_{ww} = -C''(w) + \mu G_{ww}(v,w) < 0.$

Thus, the Hessian matrix is

$$D^2\mathcal{H}(q,f,w) = \begin{pmatrix} \mathcal{H}_{qq} & 0 & 0 \\ 0 & \mathcal{H}_{ff} & \mathcal{H}_{fw} \\ 0 & \mathcal{H}_{fw} & \mathcal{H}_{ww} \end{pmatrix}.$$

Let M_1, M_2, M_3 be the main minors. The matrix is negative semi-definite if $M_1 \leq 0$, $M_2 \geq 0$, and $M_3 \leq 0$. Specifically, $M_1 = \mathcal{H}_{qq} < 0; M_2 = \mathcal{H}_{qq}\mathcal{H}_{ff} > 0;;\ M_3 = \mathcal{H}_{qq}(\mathcal{H}_{ff}\mathcal{H}_{ww} - \mathcal{H}^2_{fw})$, with $\mathcal{H}_{ff}\mathcal{H}_{ww} - \mathcal{H}^2_{fw} = (s^2 B^a_{aa}(A-sf) + \mu H_{ff}(f)G_v(v,w))\mathcal{H}_{ww} - C''(w)\mu H_f^2 G_{vv}(v,w) + \mu^2 H_f(f)^2 \triangle$, where $\triangle = (G_{vv}(v,w)G_{ww}(v,w) - G^2_{vw}(v,w))$. Since $G(v,w)$ is homogeneous of degree one, then $\triangle = 0$. It follows that:

$$\mathcal{H}_{ff}\mathcal{H}_{ww} - \mathcal{H}_{fw}^2 = (s^2 B_{aa}^a (A - sf) + \mu H_{ff}(f) G_v(v,w))\mathcal{H}_{ww} - C''(w)\mu H_f^2 G_{vv}(v,w) > 0,$$

which implies $M_3 < 0$. Consequently, the Hessian matrix is negative semi-definite, and the Hamiltonian $\mathcal{H}(q, f, w)$ is concave in (q, f, w). The determination of the optimal trajectory of q, f, and w leads to the optimal recycling rate path $\delta = w/(q + \bar{x})$.

GMM Estimation

Recall that the model's equilibrium conditions are fully characterized by the moment conditions given by

$$\mathbb{E}_t[\boldsymbol{m}(\boldsymbol{z}_t, \boldsymbol{\beta})] = 0 \tag{A.6}$$

where the components of $\boldsymbol{m}(\boldsymbol{z}_t, \boldsymbol{\beta})$ are built from the structural system (28) - (31) using instruments. These instruments include lagged annual changes in the prices of paper and recovered paper, which are strongly correlated with all other variables in the model. This means the vector of instruments is $[1,\ \Delta p_{t-1},\ \Delta p_{t-1}^w]$. Hence, by multiplying the structural equations with each of these instruments, we generate $L = 12$ moment restrictions that we use to estimate the model parameters. Given time series data of size n, the GMM estimator $\widehat{\boldsymbol{\beta}}$ is then obtained by minimizing the criterion:

$$\boldsymbol{Q_n}(\boldsymbol{\beta}) = \left[\sum_{t=1}^{n} \frac{\boldsymbol{m}(\boldsymbol{z}_t, \boldsymbol{\beta})}{n}\right]' \Omega \left[\sum_{t=1}^{n} \frac{\boldsymbol{m}(\boldsymbol{z}_t, \boldsymbol{\beta})}{n}\right]$$

where Ω is a positive definite weighting matrix that we take to be the identity, so that all moment conditions have equal weights. The variance-covariance matrix of the model is given by

$$\text{var}\left[\boldsymbol{m}(\boldsymbol{z}_t, \widehat{\boldsymbol{\beta}})\right] = \frac{1}{n}\sum_{t=1}^{n} \boldsymbol{m}(\boldsymbol{z}_t, \widehat{\boldsymbol{\beta}})\boldsymbol{m}'(\boldsymbol{z}_t, \widehat{\boldsymbol{\beta}})$$

and is used to compute the standard errors of the model's parameter estimates for efficiency.

To evaluate the model's goodness of fit, we use Sargan's J-statistic, which assesses the overall fit of the GMM estimation by testing whether the moment conditions defined in Equation (A.6) are jointly valid. This statistic is defined by

$$J_n = n \left[\sum_{t=1}^{n} \frac{\boldsymbol{m}(\boldsymbol{z}_t, \widehat{\boldsymbol{\beta}})}{n}\right]' \left[\text{var}[\boldsymbol{m}(\boldsymbol{z}_t, \widehat{\boldsymbol{\beta}})]\right]^{-1} \left[\sum_{t=1}^{n} \frac{\boldsymbol{m}(\boldsymbol{z}_t, \widehat{\boldsymbol{\beta}})}{n}\right],$$

and converges to a χ_2 distribution with degrees of freedom equal to the number of moments minus the number of parameters (which is 4 in this case) under the null hypothesis of the validity of the moment conditions. Large values of the statistic J_n provide evidence against the null.

Graph of productions of paper and logging industries

Figure A.1: Production of paper: Curve fitting

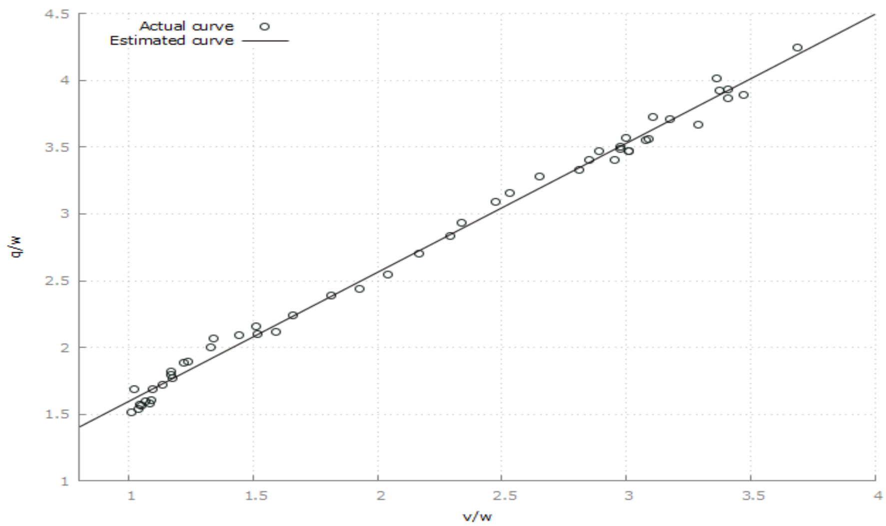
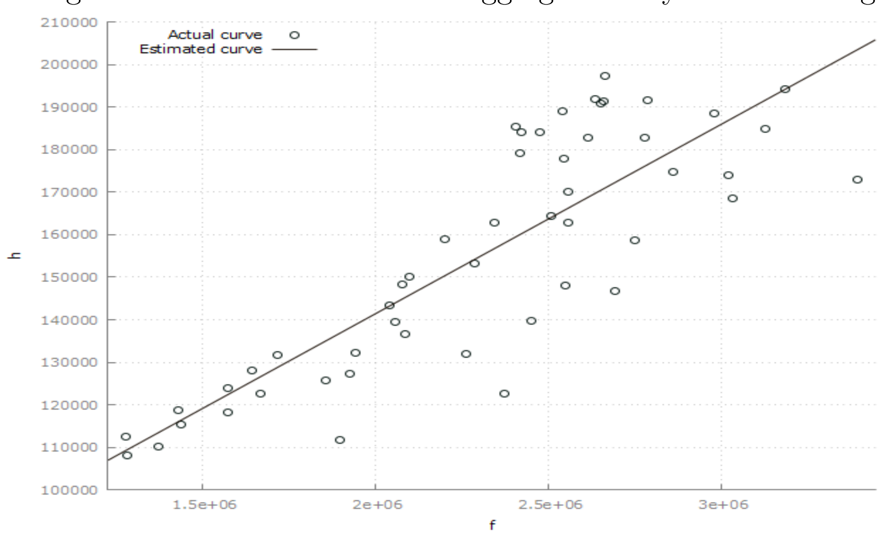


Figure A.2: Production of the logging industry: Curve fitting



Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

During the preparation of this work, the authors used Gemini (Google) in refining the framing and linguistic clarity of the the manuscript. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the final content of the publication.

References

[1] Barbier, Edward B. and Joanne C. Burgess (1997), "The Economics of Tropical Forest Land Use Options", *Land Economics*, 73(2): 174–95.

[2] Berck, Peter (1979), "The economics of timbers: A renewable resource in the long run", *Bell Journal of Economics*, 10: 447–62.

[3] Bongers, Aneli and P. Casas (2022), "The circular economy and the optimal recycling rate: A macroeconomic approach", *Ecological Economics*, 199: 107504.

[4] Brazee, Richard and Robert Mendelsohn (1990), "A dynamic model of timber markets", *Forest Science*, 36: 255–64.

[5] Chaudhuri, S. (2019) "Recycling Rethink: What to Do with Trash Now That China Won't Take It", *The Wall Street Journal*, December 19, 2019.

[6] Chaudhuri, S. (2024) "The 100-Year Quest to Make a Paper Bottle", *The Wall Street Journal*, May 15, 2024.

[7] Darby, Michael R. (1973), "Paper Recycling and the Stock of Trees", *Journal of Political Economy* 81: 1253–1255.

[8] EPA (2020), Recycling Economic Information (REI) Report 2020. US Environmental Protection Agency.

[9] Fabbri, G., Faggian, S and G. Freni (2015), "On the Mitra-Wan forest management problem in continuous time", Journal of Economic Theory 157, 1001–1040.

[10] Faustmann, M., 1849. "On the Determination of the Value which Forest Land and Immature Stands Possess for Forestry", reprinted. In: Gane, M. (Ed.), Martin Faustmann and the Evolution of Discounted Cash Flow (In En&H, 1968). Oxford Institute Paper.

[11] Hook, L. and J. Reed (2018). "Why the world's recycling system stopped working". *Financial Times*

[12] Howard, James L. and Liang, Shaobo. 2019. "U.S. timber production, trade, consumption, and price statistics, 1965-2017". Research Paper FPL-RP-701. Madison, WI: U.S. Department of Agriculture, Forest Service.

[13] Ince, P. (1995), "What Won't Get Harvested Where and When: The Effect of Increased Paper Recycling on Timber Harvest", Program on Solid Waste Policy, Working Paper.

[14] Kinnaman Thomas C., T. Shinkuma, and M. Yamamoto (2014), "The Socially Optimal Recycling Rate: Evidence from Japan", *Journal of Environmental Economics and Management*, 68: 54–70.

[15] Kinnaman, Thomas C. (2010), "The Costs of Municipal Curbside Recycling and Waste Collection", *Resources,Conservation and Recycling*, (2010) : 864-871.

[16] Kooten, V.C., Binkley, C.S., Delcourt, G. (1995), "Effect of carbon taxes and subsidies on optimal forest rotation age and supply of carbon services", *American Journal of Agricultural Economic*, 77: 365-374.

[17] Lafforgue, G. and E. Lorang (2022): "Recycling under environmental, climate and resource, constraints", *Resource and Energy Economics* 67: 101278

[18] Lafforgue, G. and L. Rouge (2019): "A dynamic model of recycling with endogenous technological breakthrough", *Resource and Energy Economics* 57: 101-118.

[19] Lopez, Rigoberto A., Farhed A. Shah and Marilyn A. Altobello (1994), "Amenity Benefits and the Optimal Allocation of Land", *Land Economics* 70: 53–62.

[20] Mendelsohn Richard and Sohngen Brent (2019), "The net carbon emissions from historical land use and land use change", *Journal of Forest economics*, 34: 263–83.

[21] Mitra, T. and Wan, H.Y. (1985). "Some theoretical results on the economics of forestry", *Review of Economic Studies*, 52 (2): 263–282.

[22] Mitra, T. and Wan, H.Y. (1986), "On the Faustmann solution to the forest management problem", *Journal of Economic Theory*, 40: 229–249.

[23] Oswalt, Sonja N., W. Brad Smith, Patrick D. Miles, and Scott A. Pugh. (2019), "Forest Resources of the United States, 2017: A Technical Document Supporting the Forest Service 2020 RPA Assessment", United States Department of Agriculture, Forest Service.

[24] Parajuli,R., S. J. Chang and R. C. Hill (2015), "How Effective Is the United States-Canada Softwood Lumber Agreement 2006? An Econometric Study", *Forest Science*, 61(6): 1041–1049.

[25] Poore, D., P. Burgess, J. Palmer, S. Rietbergen, and T. Synnott. (1989). "No Timber Without Trees". London: Earthscan Publications.

[26] Salo, S. and Tahvonen, O. (2003), "On the economics of forest vintages", *Journal of Economic Dynamics & Control*, 27: 1411–1435.

[27] Salo, S. and Tahvonen, O. (2004), "Renewable resources with endogenous age classes and allocation of land", *American Journal of Agricultural Economics*, 86(2):513— 530.

[28] Samuelson, P. 1976. "Economics of Forestry in an Evolving Society." Economic Inquiry 14:466-92

[29] Sedjo, Roger A., and Kenneth S. Lyon (1990), "The long-term adequacy of world timber supply", Resource for the Future. Baltimore: The Johns Hopkins University.

[30] Sohngen, Brent and Robert Mendelsohn (1998), "Valuing the impact of large-scale ecological change in a market: The effect of climate change on U.S. timber", *American Economic Review*, 88 :686–710.

[31] Sydsaeter, K., Hammond P. and A. Strom, (2012) "Essential Mathematics for Economic Analysis", fourth edition, Pearson.

[32] Tatoutchoup, F. D., (2016)," Optimal rate of paper recycling", *Forest Policy and Economics*, 73: 264–269.

[33] Tatoutchoup, D. and Gaudet G. (2010),"The impact of recycling on the long-run forestry", *Canadian Journal of Economics* 44: 804–813.

[34] Tita, B. (2018) "Recycling, Once Embraced by Businesses and Environmentalists, Now Under Siege", *The Wall Street Journal*, May 13, 2018.

[35] The Economist (2007) "The price of virtue: How to get people recycling more-even if they do not particularly want to", The Economist; London Vol. 383, NÂ° 8532, (Jun 9, 2007): 12.

[36] Wiseman Clark (1993), "Increased U.S. Wastepaper Recycling: the Effect on Timber Demand", *Resources, Conservation and Recycling*, 8: 103–11.

[37] Yamamoto, M. and T.C. Kinnaman (2022): "Is incineration repressing recycling?" *Journal of Environmental Economics and Management* 111: 102593